

IB Maths AA HL Unit 5 Calculus Practice Exam: Mark Scheme

IB Mathematics: Analysis and Approaches HL · 10 questions · 58 marks

- M marks are for a correct method; A marks for an accurate answer that follows a correct method; AG means the answer is given in the question, so the working must be complete.
- Accept any correct method that reaches a correct answer, unless the question says which method to use.
- Practise these topics online with instant feedback: ibmathrevision.com/HLAAUnit5

Section A: no calculator

Question 1

3 marks

$$f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - (x+h) - 3x^2 + x}{h} \text{ M1}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - h}{h} \text{ A1}$$

$$= \lim_{h \rightarrow 0} (6x + 3h - 1) = 6x - 1 \text{ A1}$$

Question 2

4 marks

- a. $3x^2 - 12$ **A1**
- b. $3x^2 - 12 < 0$ **M1**
- $-2 < x < 2$ **A1A1**

Question 3

10 marks

- a. $a + b = -1$ **A1**
- $3a + 2b = 0$ **A1**
- b. solving **(M1)**
- $a = 2, b = -3$ **A1A1**
- c. $6x^2 - 6x = 0$ **(M1)**
- $(0, 0)$ **A1**
- d. $y(2) = 16 - 12 = 4$, gradient $24 - 12 = 12$ **A1A1**
- $y = 12x - 20$ **A1**

Question 4

3 marks

$$\int (1 - 2x^{-2}) dx \text{ M1}$$

$$x + 2x^{-1} + c \text{ A1A1}$$

Question 5

4 marks

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 - (x+h)^2}{h x^2 (x+h)^2} \text{ M1A1}$$

$$= \frac{-2x - h}{x^2 (x+h)^2} \text{ A1}$$

as $h \rightarrow 0$: $\frac{-2x}{x^4} = -\frac{2}{x^3}$ **M1AG**



Question 6**8 marks**

- a. $\frac{d}{dx} \arctan(2x) = \frac{2}{1+4x^2}$ **A1**
 $f'(x) = \arctan(2x) + \frac{2x}{1+4x^2}$ **M1A1**
- b. $\arctan 1 + \frac{1}{2}$ **(M1)**
 $= \frac{\pi}{4} + \frac{1}{2}$ **A1**
- c. $3^x \ln 3 \sec x + 3^x \sec x \tan x$ **M1A1A1**

Question 7**8 marks**

- a. $\frac{dy}{dx} = 2x - 4 = 2$ at $x = 3$ **M1A1**
 $y = 2x - 2$ **A1**
- b. gradient $-\frac{1}{2}$ **(A1)**
 $y = -\frac{1}{2}x + \frac{11}{2}$ **A1**
- c. $x^2 - 4x + 7 = -\frac{1}{2}x + \frac{11}{2}$ **M1**
 $2x^2 - 7x + 3 = 0 \implies x = \frac{1}{2}$ **A1**
 $Q\left(\frac{1}{2}, \frac{21}{4}\right)$ **A1**

Question 8**8 marks**

- a. $\frac{d}{dx} \sqrt{2x+1} = \frac{1}{\sqrt{2x+1}}$ **A1**
 $h'(x) = \sqrt{2x+1} + \frac{x}{\sqrt{2x+1}}$ **M1A1**
 $= \frac{2x+1+x}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}}$ **AG**
- b. $3x+1=0$ **(M1)**
 $x = -\frac{1}{3}$ **A1**
- c. $h(4) = 12, h'(4) = \frac{13}{3}$ **A1A1**
 $y - 12 = \frac{13}{3}(x - 4)$ **A1**

Question 9**7 marks**

- a. $1+a=b-1$ **M1**
 $b=a+2$ **A1**
- b. left derivative $2x=2$, right derivative b **M1**
 $b=2, a=0$ **A1A1**
- c. $h \rightarrow 0^-: \frac{(1+h)^2-1}{h} = 2+h \rightarrow 2$ **A1**
 $h \rightarrow 0^+: \frac{2(1+h)-1-1}{h} = 2$ **A1**



Section B: calculator allowed

Question 10

3 marks

- a. 2.55 **A2**
- b. the area between the graph of f , the x -axis and the line $x = 4$ **A1**

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