

IB Maths AA HL Unit 3 Geometry and Trigonometry Practice Exam

IB Mathematics: Analysis and Approaches HL · 11 questions · 60 marks

Name _____

Date _____

Score / 60

- Answer every question. Show your working: most marks are for method.
- Section A no calculator; Section B calculator allowed.
- Topics: 3D Geometry Problems; Trigonometric Identities and Equations; Vector Definitions and Operations; Vector Angles; Vector Line Equations and Intersection; Vector Product Applications.
- The mark scheme is a separate free PDF: ibmathrevision.com/downloads/practice-exams/hlaa/ib-maths-aa-hl-unit-3-geometry-trigonometry-practice-exam-mark-scheme.pdf

Section A: no calculator

1. Find the coordinates of the point where the line $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ meets the plane $x + 2y + 3z = 13$. [3]

2. The graph of $y = \sin x$, $0 \leq x \leq 2\pi$, is symmetric about the line $x = \frac{\pi}{2}$ on the interval $0 \leq x \leq \pi$. [4]
- a. Use this symmetry to write down an identity for $\sin(\pi - x)$. [1]
- b. Given that $\sin \frac{\pi}{5} = s$, express $\sin \frac{4\pi}{5}$, $\sin \frac{6\pi}{5}$ and $\sin \frac{9\pi}{5}$ in terms of s . [3]

3. $ABCD$ is a parallelogram with $A(1, 0, 2)$, $B(4, 1, 5)$ and $C(6, -2, 3)$. [4]
- a. Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$. [2]
- b. Find the coordinates of D . [2]



4. Find the acute angle between the lines with direction vectors $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$, giving your answer exactly. [4]

5. The lines $L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ are given. [5]

- a. Show that L_1 and L_2 are not parallel. [1]
b. Determine whether L_1 and L_2 intersect or are skew. [4]

6. Use the symmetry properties of the cosine function to answer the following. [4]

- a. Show that $\cos(\pi - \theta) + \cos \theta = 0$. [1]
b. Hence find the exact value of $\sum_{r=0}^{11} \cos\left(\frac{r\pi}{6}\right)$. [3]

7. Points A and B have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$. [8]

- a. Find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$. [2]
b. The point R lies on $[AB]$ with $AR : RB = 1 : 3$. Find the position vector of R . [2]
c. The point S has position vector $\mathbf{a} + \lambda \overrightarrow{AB}$. Find the value of λ for which $|\overrightarrow{OS}|$ is least. [4]



8. Prove, using the scalar product, that the angle in a semicircle is a right angle: if O is the centre of a circle, $[AB]$ a diameter with $\vec{OA} = \mathbf{a}$, $\vec{OB} = -\mathbf{a}$, and P a point on the circle with $\vec{OP} = \mathbf{p}$, show that $\vec{AP} \cdot \vec{BP} = 0$. [5]

9. The points $A(2, 0, 1)$, $B(1, 3, 0)$ and $C(0, 1, 4)$ are given, and O is the origin. [7]

a. Find $\vec{AB} \times \vec{AC}$. [3]

b. Find the exact area of triangle ABC . [2]

c. Find the volume of the tetrahedron $OABC$, given that it equals $\frac{1}{6} \left| \vec{OA} \cdot (\vec{OB} \times \vec{OC}) \right|$. [2]

10. Three planes have equations $x + y + z = 4$, $2x - y + 3z = 7$ and $x - 2y + kz = m$, where $k, m \in \mathbb{R}$. [8]

a. Find the value of k for which the planes do not meet at a single point. [3]

b. For this value of k , find the value of m for which the planes meet in a line. [2]

c. For these values, find a vector equation of the line. [3]



Section B: calculator allowed

11. The planes $\Pi_1 : x - 2y + 2z = 3$ and $\Pi_2 : 2x + y - 2z = 1$ are given. [8]

- a. Find the acute angle between the planes. [3]
- b. Find a vector equation of the line of intersection of the planes. [5]

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