

IB Maths AA HL Unit 3 Geometry and Trigonometry Practice Exam: Mark Scheme

IB Mathematics: Analysis and Approaches HL · 11 questions · 60 marks

- M marks are for a correct method; A marks for an accurate answer that follows a correct method; R marks are for clear reasoning.
- Accept any correct method that reaches a correct answer, unless the question says which method to use.
- Practise these topics online with instant feedback: ibmathrevision.com/HLAAUnit3

Section A: no calculator

Question 1

3 marks

$$(1 + 2t) + 2t + 3(2 - t) = 13 \text{ M1}$$

$$t = 6 \text{ A1}$$

$$(13, 6, -4) \text{ A1}$$

Question 2

4 marks

$$\text{a. } \sin(\pi - x) = \sin x \text{ A1}$$

$$\text{b. } \sin \frac{4\pi}{5} = s \text{ A1}$$

$$\sin \frac{6\pi}{5} = -s \text{ A1}$$

$$\sin \frac{9\pi}{5} = -s \text{ A1}$$

Question 3

4 marks

$$\text{a. } \overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} \text{ A1}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} \text{ A1}$$

$$\text{b. } \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC} \text{ (M1)}$$

$$D(3, -3, 0) \text{ A1}$$

Question 4

4 marks

$$\mathbf{d}_1 \cdot \mathbf{d}_2 = 2 - 4 - 2 = -4 \text{ A1}$$

$$\cos \theta = \frac{|-4|}{3 \times 3} = \frac{4}{9} \text{ M1A1}$$

$$\theta = \arccos \frac{4}{9} \text{ A1}$$



Question 5

5 marks

- a. direction vectors are not scalar multiples **R1**
- b. $1 + \lambda = 4 + 2\mu$, $2 - \lambda = -2 + \mu$ **M1**
 $\lambda = \frac{11}{3}$, $\mu = \frac{1}{3}$ **A1**
 third component: $3 + 2\lambda = \frac{31}{3}$, $5 - \mu = \frac{14}{3}$ **M1**
 not equal, so the lines are skew **A1**

Question 6

4 marks

- a. $\cos(\pi - \theta) = -\cos \theta$ **M1AG**
- b. pair r with $6 - r$ for $r = 1, \dots, 5$ and r with $18 - r$ for $r = 7, \dots, 11$: pairs cancel **M1**
 remaining terms $\cos 0 + \cos \pi = 1 - 1$ **A1**
 sum = 0 **A1**

Question 7

8 marks

- a. $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$ **A1**
 $|\overrightarrow{AB}| = \sqrt{4 + 16 + 16} = 6$ **A1**
- b. $\mathbf{r} = \mathbf{a} + \frac{1}{4}\overrightarrow{AB}$ **M1**
 $= \begin{pmatrix} \frac{7}{2} \\ 1 \\ 0 \end{pmatrix}$ **A1**
- c. $|\overrightarrow{OS}|^2 = (3 + 2\lambda)^2 + (2 - 4\lambda)^2 + (-1 + 4\lambda)^2$ **M1**
 $= 36\lambda^2 - 12\lambda + 14$ **A1**
 minimum at $\lambda = \frac{1}{6}$ **M1A1**

Question 8

5 marks

- $\overrightarrow{AP} = \mathbf{p} - \mathbf{a}$, $\overrightarrow{BP} = \mathbf{p} + \mathbf{a}$ **A1A1**
 $(\mathbf{p} - \mathbf{a}) \cdot (\mathbf{p} + \mathbf{a}) = |\mathbf{p}|^2 - |\mathbf{a}|^2$ **M1A1**
 $|\mathbf{p}| = |\mathbf{a}|$ (radii), so the product is 0 and $\hat{APB} = 90^\circ$ **R1AG**

Question 9

7 marks

- a. $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ **A1**
 $\begin{pmatrix} 10 \\ 5 \\ 5 \end{pmatrix}$ **M1A1**
- b. $\frac{1}{2}\sqrt{150}$ **M1A1**
- c. $\overrightarrow{OB} \times \overrightarrow{OC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ **A1**
 volume = $\frac{25}{6}$ **A1**



Question 10

8 marks

a. row-reduce: $-3y + z = -1$ and $-3y + (k - 1)z = m - 4$ **M1A1**

$$k - 1 = 1 \implies k = 2 \text{ **A1**}$$

b. $m - 4 = -1$ **M1**

$$m = 3 \text{ **A1**}$$

c. let $z = t$: $y = \frac{1+t}{3}$, $x = 4 - y - z = \frac{11-4t}{3}$ **M1A1**

$$\mathbf{r} = \begin{pmatrix} 11/3 \\ 1/3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4/3 \\ 1/3 \\ 1 \end{pmatrix} \text{ **A1**}$$

Section B: calculator allowed**Question 11**

8 marks

a. $\cos \theta = \frac{|2-2-4|}{3 \times 3}$ **M1A1**

$$\theta = 63.6^\circ \text{ **A1**}$$

b. direction $\mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}$ **M1A1**a common point, e.g. $z = 0$: $x - 2y = 3$, $2x + y = 1 \implies (1, -1, 0)$ **M1A1**

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix} \text{ **A1**}$$

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