



★ not in the IB formula booklet — learn it. Everything else is printed in the booklet (its notation may differ slightly).

Prior learning

Area: parallelogram, triangle, trapezoid:

$$A = bh, A = \frac{1}{2}bh, A = \frac{1}{2}(a+b)h$$

Circle: $A = \pi r^2, C = 2\pi r$

Volume: cuboid, cylinder, prism: $V = lwh$,
 $V = \pi r^2 h, V = Ah$

Curved surface of a cylinder: $A = 2\pi rh$

Distance; midpoint:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2},$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

★ Pythagoras: $a^2 + b^2 = c^2$

★ Right-angled trig: $\sin \theta = \frac{\text{opp}}{\text{hyp}},$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}, \tan \theta = \frac{\text{opp}}{\text{adj}}$$

★ Laws of indices: $a^m a^n = a^{m+n},$
 $\frac{a^m}{a^n} = a^{m-n}, (a^m)^n = a^{mn}, a^{-n} = \frac{1}{a^n}$

1 Number & algebra

★ Standard form: $a \times 10^k,$

$$1 \leq a < 10, k \in \mathbb{Z}$$

Arithmetic sequence: $u_n = u_1 + (n-1)d$

Arithmetic series:

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$$

★ Common difference: $d = u_{n+1} - u_n$

Geometric sequence: $u_n = u_1 r^{n-1}$

Geometric series, $r \neq 1$: $S_n = \frac{u_1(r^n - 1)}{r - 1}$
 $= \frac{u_1(1 - r^n)}{1 - r}$

Compound interest (k times a year, n years, $r\%$): $FV = PV(1 + \frac{r}{100k})^{kn}$

★ Depreciation at $r\%$ a year:

$$V = V_0(1 - \frac{r}{100})^n$$

Loans, annuities, amortisation: use the GDC's TVM solver ($N, I\%, PV, PMT, FV, P/Y, C/Y$); money paid out is negative.

Exponents & logs: $a^x = b \iff x = \log_a b$

★ Log facts: $\log_{10} 10^x = x, \ln e^x = x,$
 $e^{\ln x} = x$

Percentage error: $\epsilon = \left| \frac{v_A - v_B}{v_B} \right| \times 100\%$

★ Bounds: a value rounded to the nearest u lies in $[x - \frac{u}{2}, x + \frac{u}{2})$

2 Functions & models

Gradient; forms of a line: $m = \frac{y_2 - y_1}{x_2 - x_1},$

$$y = mx + c, ax + by + d = 0,$$

$$y - y_1 = m(x - x_1)$$

★ Parallel; perpendicular: $m_1 = m_2;$

$$m_1 m_2 = -1$$

Quadratic: axis of symmetry: $x = -\frac{b}{2a}$

★ Linear; quadratic models:

$$f(x) = mx + c; f(x) = ax^2 + bx + c$$

★ Exponential models: $f(x) = ka^x + c,$
 $f(x) = ka^{-x} + c, f(x) = ke^{rx} + c$

★ Direct / inverse variation: $f(x) = ax^n,$
 $n \in \mathbb{Z}$

★ Cubic model:

$$f(x) = ax^3 + bx^2 + cx + d$$

★ Sinusoidal model (degrees):

$$f(x) = a \sin(bx) + d : \text{amplitude } |a|,$$

$$\text{period } \frac{360^\circ}{b}$$

★ Inverse function: swap x and y ; graph reflects in $y = x$; domain of f^{-1} = range of f

★ Horizontal asymptote of $ka^x + c$: $y = c$

3 Geometry & trigonometry

3D distance ($\Delta x = x_1 - x_2, \dots$):

$$d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

3D midpoint: $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2})$

Pyramid; cone: $V = \frac{1}{3}Ah; V = \frac{1}{3}\pi r^2 h,$

curved $A = \pi rl$

Sphere: $V = \frac{4}{3}\pi r^3, A = 4\pi r^2$

★ Slant height of a cone: $l = \sqrt{r^2 + h^2}$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C,$
 $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Area of a triangle: $A = \frac{1}{2}ab \sin C$

Arc; sector (θ in degrees): $l = \frac{\theta}{360} \times 2\pi r,$
 $A = \frac{\theta}{360} \times \pi r^2$

★ Elevation / depression measured from the horizontal; bearings clockwise from north, 3 figures

★ Perpendicular bisector of AB : through the midpoint of AB , gradient $-\frac{1}{m_{AB}}$

★ Voronoi: cell edges lie on perpendicular bisectors of pairs of sites; the point furthest from every site ("toxic waste dump") is at a vertex or on the boundary

4 Statistics & probability

Interquartile range: $IQR = Q_3 - Q_1$

Mean ($n = \sum f_i$): $\bar{x} = \frac{\sum f_i x_i}{n}$

★ Outliers: $x < Q_1 - 1.5 IQR$ or
 $x > Q_3 + 1.5 IQR$

★ Standard deviation: $\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{n}}$

★ Data $\times a$ then $+b$: mean $\rightarrow a\bar{x} + b,$
s.d. $\rightarrow |a|\sigma$

★ Regression: $y = ax + b$ (GDC); use to predict y within the data range only

★ $|r|$ near 1: strong linear correlation;
Spearman's r_s = PMCC of the ranks

Probability: $P(A) = \frac{n(A)}{n(U)},$
 $P(A) + P(A') = 1$

★ Expected number of occurrences:
 $n \times P(A)$

Combined events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

Independent: $P(A \cap B) = P(A)P(B)$

Conditional: $P(A | B) = \frac{P(A \cap B)}{P(B)}$

Expected value: $E(X) = \sum x P(X = x)$

★ Valid distribution; fair game:

$$\sum P(X = x) = 1; E(\text{gain}) = 0$$

★ Binomial $X \sim B(n, p)$:

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$$

Binomial mean, variance: $E(X) = np,$

$$\text{Var}(X) = np(1 - p)$$

★ Normal: about 68%, 95%, 99.7% of values lie within 1, 2, 3 s.d. of μ

χ^2 statistic: $\chi^2_{\text{calc}} = \sum \frac{(f_o - f_e)^2}{f_e}$

★ Expected frequency (independence):

$$f_e = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

★ Degrees of freedom:

independence: $(r - 1)(c - 1);$

goodness of fit: classes $- 1$

★ Test decision: reject H_0 if p -value < significance level (or $\chi^2_{\text{calc}} > \text{critical value}$)

★ t -test (GDC): compares two population means; $H_0: \mu_1 = \mu_2$

5 Calculus

Derivative of x^n : $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

★ Constant multiples, sums:

$$f(x) = ax^n + bx^m \Rightarrow f'(x) = anx^{n-1} + bmx^{m-1}$$

★ Gradient of tangent at $x = a$: $f'(a);$

tangent $y - f(a) = f'(a)(x - a)$

★ Gradient of normal: $-\frac{1}{f'(a)}$

★ Increasing / decreasing: $f'(x) > 0;$
 $f'(x) < 0$

★ Stationary points; optimisation:

$f'(x) = 0$; check the sign of f' either side

Integral of $x^n, n \neq -1$: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Area, $y \geq 0$: $A = \int_a^b y dx$

Trapezoidal rule, $h = \frac{b-a}{n}$: $\int_a^b y dx \approx \frac{1}{2}h[(y_0 + y_n) + 2(y_1 + \dots + y_{n-1})]$

★ Boundary condition: find C by substituting a known point into $\int f'(x) dx$