FORMULAS YOU MUST MEMORISE

IB Mathematics: Applications & Interpretation HL

Not in the IB formula booklet: learn these by heart.

★ Not given in the exam: learn every formula on this page. The full sheet and the booklet have the rest.

Prior learning

★ **Pythagoras; SOHCAHTOA; indices:** $a^2 + b^2 = c^2$; $\tan\theta = \frac{\text{opp}}{\text{adj}}$; $a^m a^n = a^{m+n}$
When: Right-angled triangles, elevation and depression

1 Number & algebra

★ **Standard form:** $a \times 10^k$, $1 \le a < 10$
When: Read calculator notation such as $3.2E-5$

★ **Modulus, argument:** $|z| = \sqrt{a^2+b^2}$, $\arg z = \theta$, $\tan\theta = \frac{b}{a}$ (check quadrant)
When: Plot on an Argand diagram; find the modulus and argument

★ **Products:** $|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$
When: Multiply, divide and raise to powers in polar form

★ **Sinusoids:** $A\cos(\omega t) + B\cos(\omega t + \phi)$ = real part of $(A + Be^{i\phi})e^{i\omega t}$, so amplitudes and phases add as complex numbers
When: Euler form, interpreted geometrically

★ **3×3 determinant:**
$$\det A = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$
When: Determinants and inverses of larger matrices with technology

★ **Matrix rules:** $AB \neq BA$ in general, $(AB)^{-1} = B^{-1}A^{-1}$, $AX = B \Rightarrow X = A^{-1}B$
When: Multiply matrices; AB need not equal BA

★ **Eigenvalues; eigenvectors:** $\det(M - \lambda I) = 0$; $M\boldsymbol{p} = \lambda\boldsymbol{p}$
When: Characteristic polynomial, eigenvalues and eigenvectors

★ P **= eigenvectors as columns,** D **= diagonal matrix of the matching eigenvalues**
When: Write a 2×2 matrix as PDP^{-1}

2 Functions & models

★ **Perpendicular:** $m_1 m_2 = -1$
When: Finding a perpendicular line, a normal, or a perpendicular bisector

★ **Models:** $f(x) = ka^x + c$, $ke^{rx} + c$; ax^n; $a\sin(bx) + d$ (period $\frac{360°}{b}$)
When: Growth, decay, power and sinusoidal models in context

★ **Logistic model:** $f(x) = \frac{L}{1+Ce^{-kx}}$ $(L, C, k > 0)$
When: Population growth and the spread of disease

★ **Linearising:** $y = ax^b \Rightarrow \ln y = \ln a + b\ln x$; $y = ka^x \Rightarrow \ln y = \ln k + x\ln a$
When: Turning a curve into a straight line to find a and b

★ **Composite; inverse:** $(f \circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$
When: Composite functions in context; restrict a domain for an inverse

★ **Transformations:** $f(x) + b$ up; $f(x-a)$ right; $pf(x)$ vertical ×p; $f(qx)$ horizontal ×$\frac{1}{q}$
When: Apply translations, stretches and reflections in order

5 Calculus

★ **Tangent; normal:** $y - f(a) = f'(a)(x-a)$; $m_n = -\frac{1}{f'(a)}$
When: Any "find the equation of the tangent / normal" question

★ **Max / min; inflexion:** $f' = 0$, $f'' < 0$ / $f'' > 0$; $f'' = 0$ with sign change
When: Optimisation and curve sketching

★ **Related rates:** $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$
When: Link rates, such as how fast a tank's water level rises

★ **Linear inside; by inspection:** $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$; $\int g'(x)f'(g(x))\,dx = f(g(x)) + C$
When: Integrate by inspection or a simple substitution

★ **Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx$
When: Separate the variables; use the initial condition

★ **Phase portraits: real** λ **both** < 0 **stable node, both** > 0 **unstable, opposite signs saddle; complex:** $\text{Re}\,\lambda < 0$ **spiral in,** > 0 **out,** $= 0$ **closed loops**
When: Classify the equilibrium from the eigenvalues

★ **2nd order** $\ddot{x} + a\dot{x} + bx = 0$**:** put $y = \dot{x} : \dot{x} = y$, $\dot{y} = -bx - ay$
When: Second-order equations, such as a spring, as coupled systems

3 Geometry & trigonometry

★ **Radians; bearings:** π rad $= 180°$; bearings clockwise from north
When: Convert between degrees and radians; three-figure bearings

★ **Perpendicular bisectors & Voronoi: cell edges on perpendicular bisectors of sites; largest empty circle at a vertex or on the boundary**
When: Voronoi cells; nearest-neighbour interpolation

★ **Area of image:** $|\det A| \times$ area of object
When: The determinant as an area scale factor

★ **Composite transformation:** B **then** A **is** AB
When: Combine transformations

★ **Motion with constant velocity:** $\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v}$, speed $= |\boldsymbol{v}|$
When: Positions, speeds and closest approach

★ **Unit vector; perpendicular:** $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$; $\boldsymbol{v} \cdot \boldsymbol{w} = 0$
When: Unit vectors; a vector of a given length

★ **Component of** $\boldsymbol{v}$ **along** $\boldsymbol{w}$**:** $\boldsymbol{v} \cdot \hat{\boldsymbol{w}}$
When: Components of a vector in a given direction

★ **Graphs: sum of degrees =** $2\times$ **edges;** K_n **has** $\frac{1}{2}n(n-1)$ **edges; a tree on** n **vertices has** $n-1$ **edges**
When: Graph theory: counting edges; can the graph exist?

★ **Planar connected graph:** $v - e + f = 2$, $e \le 3v - 6$ $(v \ge 3)$
When: Recognise simple, complete and connected graphs

★ **Eulerian circuit: every vertex even; Eulerian trail: exactly two odd vertices**
When: Eulerian trail or circuit from vertex degrees

★ **Walks of length** k **from** i **to** j**:** $(A^k)_{ij}$
When: Adjacency matrix questions: raise A to the power k

★ **Chinese postman: add the cheapest pairing of odd vertices, repeated once each**
When: Shortest route that travels every edge and returns

★ **TSP: upper bound = nearest neighbour; lower bound = MST of the rest + two shortest edges from the deleted vertex**
When: Upper and lower bounds for the travelling salesman problem

4 Statistics & probability

★ **Outliers:** $< Q_1 - 1.5\,IQR$ **or** $> Q_3 + 1.5\,IQR$
When: Box plots and "is this value an outlier?" questions

★ **Variance:** $\text{Var}(X) = E(X^2) - [E(X)]^2$
When: Variance of a random variable

★ **Binomial pmf:** $P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$
When: Fixed number of independent trials

★ **Poisson** $X \sim \text{Po}(m)$**:** $P(X = x) = \frac{m^x e^{-m}}{x!}$
When: Random events in time or space

★ **Sum of independent Poissons:** $\text{Po}(m_1) + \text{Po}(m_2) = \text{Po}(m_1 + m_2)$
When: Combining two Poisson counts (two shops, two time periods)

★ **Sample mean (CLT,** $n \ge 30$ **or** X **normal):** $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
When: Probabilities about a sample mean

★ **Linear combinations of independent normals are normal; for** n **independent** $X_i \sim N(\mu, \sigma^2)$**:** $\sum X_i \sim N(n\mu, n\sigma^2)$
When: A linear combination of independent normals is normal

★ **Expected frequency; df:** $\frac{\text{row} \times \text{column}}{\text{total}}$; $(r-1)(c-1)$
When: Every chi-squared test for independence

★ **Goodness of fit df: classes − 1 − parameters estimated**
When: Chi-squared goodness-of-fit tests

★ **Reject** H_0 **if** p**-value < significance level. Type I: reject a true** H_0**; Type II: accept a false** H_0
When: Critical regions; Type I and Type II errors

★ **Transition matrix:** $T^n\boldsymbol{s}_0 = \boldsymbol{s}_n$**; steady state** $T\boldsymbol{s} = \boldsymbol{s}$, $\sum s_i = 1$
When: Markov chains: states after several steps; steady state

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