★ not in the IB formula booklet — learn it. Everything else is printed in the booklet (its notation may differ slightly).

Prior learning

Areas: parallelogram bh, triangle $\frac{1}{2}bh$, trapezoid $\frac{1}{2}(a+b)h$, circle πr^2

Circumference; cylinder: $C = 2\pi r$; $V = \pi r^2 h$, curved $A = 2\pi r h$

Cuboid; prism: $V = lwh$; $V = Ah$

Distance; midpoint: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

★ Pythagoras; SOHCAHTOA; indices: $a^2 + b^2 = c^2$; $\tan\theta = \frac{\text{opp}}{\text{adj}}$; $a^m a^n = a^{m+n}$

1 Number & algebra

★ Standard form: $a \times 10^k$, $1 \le a < 10$

Arithmetic: $u_n = u_1 + (n-1)d$, $S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right) = \frac{n}{2}(u_1 + u_n)$

Geometric: $u_n = u_1 r^{n-1}$, $S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$

Compound interest: $FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$

Annuities, loans, amortisation: GDC TVM solver; money paid out is negative.

Percentage error: $\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$

Exponents & logs: $a^x = b \iff x = \log_a b$

Log laws: $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^m = m \log_a x$

Change of base: $\log_b a = \frac{\log_c a}{\log_c b}$

Complex numbers: $z = a + bi = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\ \text{cis}\,\theta$

★ Modulus, argument: $|z| = \sqrt{a^2 + b^2}$, $\arg z = \theta$, $\tan\theta = \frac{b}{a}$ (check quadrant)

★ Products: $|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

★ Sinusoids: $A\cos(\omega t) + B\cos(\omega t + \phi)$ = real part of $(A + Be^{i\phi})e^{i\omega t}$, so amplitudes and phases add as complex numbers

2×2 determinant; inverse: $\det A = ad - bc$, $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

★ 3×3 determinant: $\det A = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

★ Matrix rules: $AB \ne BA$ in general, $(AB)^{-1} = B^{-1}A^{-1}$, $AX = B \Rightarrow X = A^{-1}B$

★ Eigenvalues; eigenvectors: $\det(M - \lambda I) = 0$; $M\boldsymbol{p} = \lambda\boldsymbol{p}$

Matrix powers: $M^n = PD^nP^{-1}$

★ P = eigenvectors as columns, D = diagonal matrix of the matching eigenvalues

2 Functions & models

Lines: $m = \frac{y_2 - y_1}{x_2 - x_1}$, $y = mx + c$, $ax + by + d = 0$, $y - y_1 = m(x - x_1)$

★ Perpendicular: $m_1 m_2 = -1$

Quadratic: axis of symmetry: $x = -\frac{b}{2a}$

★ Models: $f(x) = ka^x + c$, $ke^{rx} + c$; ax^n; $a\sin(bx) + d$ (period $\frac{360°}{b}$)

★ Logistic model: $f(x) = \frac{L}{1 + Ce^{-kx}}$ $(L, C, k > 0)$

★ Linearising: $y = ax^b \Rightarrow \ln y = \ln a + b\ln x$; $y = ka^x \Rightarrow \ln y = \ln k + x\ln a$

★ Composite; inverse: $(f \circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$

★ Transformations: $f(x) + b$ up; $f(x - a)$ right; $pf(x)$ vertical ×p; $f(qx)$ horizontal ×$\frac{1}{q}$

3 Geometry & trigonometry

3D distance ($\Delta x = x_1 - x_2, \ldots$): $d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

3D midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Pyramid; cone; sphere: $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, $A = \pi r l$; $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$

Sine, cosine rules; area: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, $c^2 = a^2 + b^2 - 2ab\cos C$, $A = \frac{1}{2}ab\sin C$

Arc; sector, degrees: $l = \frac{\theta}{360}2\pi r$, $A = \frac{\theta}{360}\pi r^2$

Arc; sector, radians: $l = r\theta$, $A = \frac{1}{2}r^2\theta$

Identities: $\cos^2\theta + \sin^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

★ Radians; bearings: π rad $= 180°$; bearings clockwise from north

★ Perpendicular bisectors & Voronoi: cell edges on perpendicular bisectors of sites; largest empty circle at a vertex or on the boundary

Reflection in $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Stretch ×k: horizontal, vertical; enlargement: $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$; $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$

Rotation about O by θ: anticlockwise; clockwise: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$; $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

★ Area of image: $|\det A| \times$ area of object

★ Composite transformation: B then A is AB

Vector magnitude: $|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Scalar product; angle: $\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3 = |\boldsymbol{v}||\boldsymbol{w}|\cos\theta$

Vector product: $\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$, $|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}|\sin\theta$

Area of parallelogram: $A = |\boldsymbol{v} \times \boldsymbol{w}|$

Line: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$

★ Motion with constant velocity: $\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v}$, speed $= |\boldsymbol{v}|$

★ Unit vector; perpendicular: $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$; $\boldsymbol{v} \cdot \boldsymbol{w} = 0$

★ Component of $\boldsymbol{v}$ along $\boldsymbol{w}$: $\boldsymbol{v} \cdot \hat{\boldsymbol{w}}$

★ Graphs: sum of degrees = 2× edges; K_n has $\frac{1}{2}n(n-1)$ edges; a tree on n vertices has $n - 1$ edges

★ Planar connected graph: $v - e + f = 2$, $e \le 3v - 6$ $(v \ge 3)$

★ Eulerian circuit: every vertex even; Eulerian trail: exactly two odd vertices

★ Walks of length k from i to j: $(A^k)_{ij}$

★ Chinese postman: add the cheapest pairing of odd vertices, repeated once each

★ TSP: upper bound = nearest neighbour; lower bound = MST of the rest + two shortest edges from the deleted vertex

4 Statistics & probability

IQR; mean: $IQR = Q_3 - Q_1$, $\bar{x} = \frac{\sum f_i x_i}{n}$

★ Outliers: $< Q_1 - 1.5\,IQR$ or $> Q_3 + 1.5\,IQR$

Probability: $P(A) = \frac{n(A)}{n(U)}$, $P(A) + P(A') = 1$

Combined events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Independent: $P(A \cap B) = P(A)P(B)$

Conditional: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Expected value: $E(X) = \sum x P(X = x)$

★ Variance: $\text{Var}(X) = E(X^2) - [E(X)]^2$

★ Binomial pmf: $P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$

Binomial mean, variance: np, $np(1-p)$

★ Poisson $X \sim \text{Po}(m)$: $P(X = x) = \frac{m^x e^{-m}}{x!}$

Poisson mean, variance: $E(X) = m$, $\text{Var}(X) = m$

★ Sum of independent Poissons: $\text{Po}(m_1) + \text{Po}(m_2) = \text{Po}(m_1 + m_2)$

Linear combinations: $E(a_1 X_1 \pm a_2 X_2) = a_1 E(X_1) \pm a_2 E(X_2)$

... independent X_i: $\text{Var}(a_1 X_1 \pm a_2 X_2) = a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2)$

Unbiased variance estimate: $s_{n-1}^2 = \frac{n}{n-1}s_n^2$

★ Sample mean (CLT, $n \ge 30$ or X normal): $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

★ Linear combinations of independent normals are normal; for n independent $X_i \sim N(\mu, \sigma^2)$: $\sum X_i \sim N(n\mu, n\sigma^2)$

χ^2 statistic: $\chi^2_{\text{calc}} = \sum \frac{(f_o - f_e)^2}{f_e}$

★ Expected frequency; df: $\frac{\text{row} \times \text{column}}{\text{total}}$; $(r-1)(c-1)$

★ Goodness of fit df: classes – 1 – parameters estimated

★ Reject H_0 if p-value $<$ significance level. Type I: reject a true H_0; Type II: accept a false H_0

★ Transition matrix: $T^n\boldsymbol{s}_0 = \boldsymbol{s}_n$; steady state $T\boldsymbol{s} = \boldsymbol{s}$, $\sum s_i = 1$

5 Calculus

Derivatives: $(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \frac{1}{\cos^2 x}$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$

Chain, product, quotient: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, $(uv)' = uv' + vu'$, $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

★ Tangent; normal: $y - f(a) = f'(a)(x - a)$; $m_n = -\frac{1}{f'(a)}$

★ Max / min; inflexion: $f' = 0$, $f'' < 0$ / $f'' > 0$; $f'' = 0$ with sign change

★ Related rates: $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

Integrals: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $\int \frac{1}{x}\,dx = \ln|x| + C$, $\int \sin x\,dx = -\cos x + C$

More integrals: $\int \cos x\,dx = \sin x + C$, $\int \frac{1}{\cos^2 x}\,dx = \tan x + C$, $\int e^x\,dx = e^x + C$

★ Linear inside; by inspection: $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$; $\int g'(x)f'(g(x))\,dx = f(g(x)) + C$

Areas: $\int_a^b |y|\,dx$, $\int_c^d |x|\,dy$

Volumes of revolution: $V = \int_a^b \pi y^2\,dx$, $V = \int_c^d \pi x^2\,dy$

Trapezoidal rule, $h = \frac{b-a}{n}$: $\int_a^b y\,dx \approx \frac{1}{2}h\left[(y_0 + y_n) + 2(y_1 + \cdots + y_{n-1})\right]$

Kinematics: $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$; distance $\int_{t_1}^{t_2} |v|\,dt$

★ Separable: $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx$

Euler's method: $y_{n+1} = y_n + h\,f(x_n, y_n)$, $x_{n+1} = x_n + h$

Euler, coupled systems: $x_{n+1} = x_n + h\,f_1(x_n, y_n, t_n)$, $y_{n+1} = y_n + h\,f_2(x_n, y_n, t_n)$, $t_{n+1} = t_n + h$

Coupled linear system $\dot{\boldsymbol{x}} = M\boldsymbol{x}$: $\boldsymbol{x} = Ae^{\lambda_1 t}\boldsymbol{p}_1 + Be^{\lambda_2 t}\boldsymbol{p}_2$

★ Phase portraits: real λ both < 0 stable node, both > 0 unstable, opposite signs saddle; complex: $\text{Re}\,\lambda < 0$ spiral in, > 0 out, $= 0$ closed loops

★ 2nd order $\ddot{x} + a\dot{x} + bx = 0$: put $y = \dot{x}$: $\dot{x} = y$, $\dot{y} = -bx - ay$

2021 syllabus · v1 Sept 2026