IB Math Revision
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FORMULA POSTER

IB Mathematics: Applications & Interpretation HL

2021 syllabus · v2 Oct 2026

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formula-sheet-hlai
Interactive version

★ Learn it: not in the IB formula booklet 📖 In the booklet

Prior learning

📖 Areas: parallelogram bh, triangle $\frac{1}{2}bh$, trapezoid $\frac{1}{2}(a+b)h$, circle πr^2

📖 Circumference; cylinder: $C = 2\pi r$; $V = \pi r^2 h$, curved $A = 2\pi rh$

📖 Cuboid; prism: $V = lwh$; $V = Ah$

Scan to practise

📖 Distance; midpoint: $d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

★ **Pythagoras; SOHCAHTOA; indices:** $a^2+b^2=c^2$; $\tan\theta = \frac{\text{opp}}{\text{adj}}$; $a^m a^n = a^{m+n}$ → Right-angled triangles, elevation and depression

1 Number & algebra

★ **Standard form:** $a \times 10^k$, $1 \le a < 10$ → Read calculator notation such as $3.2E-5$

Scan to practise

Watch for: Convert calculator E-notation to $a \times 10^n$

📖 Arithmetic: $u_n = u_1 + (n-1)d$, $S_n = \frac{n}{2}(2u_1+(n-1)d) = \frac{n}{2}(u_1+u_n)$

📖 Geometric: $u_n = u_1 r^{n-1}$, $S_n = \frac{u_1(r^n-1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$

📖 Sum to infinity, $|r|<1$: $S_\infty = \frac{u_1}{1-r}$

📖 Compound interest: $FV = PV\left(1+\frac{r}{100k}\right)^{kn}$

Annuities, loans, amortisation: GDC TVM solver; money paid out is negative.

📖 Percentage error: $\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$

📖 Exponents & logs: $a^x = b \iff x = \log_a b$

📖 Log laws: $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^m = m\log_a x$

📖 Change of base: $\log_b a = \frac{\log_c a}{\log_c b}$

📖 Complex numbers: $z = a+bi = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\ \text{cis}\,\theta$

★ **Modulus, argument:** $|z| = \sqrt{a^2+b^2}$, $\arg z = \theta$, $\tan\theta = \frac{b}{a}$ (check quadrant) → Plot on an Argand diagram; find the modulus and argument

Watch for: Work in radians for arguments

★ **Products:** $|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ → Multiply, divide and raise to powers in polar form

★ **Sinusoids:** $A\cos(\omega t) + B\cos(\omega t + \phi)$ = real part of $(A+Be^{i\phi})e^{i\omega t}$, so amplitudes and phases add as complex numbers → Euler form, interpreted geometrically

📖 2×2 determinant; inverse: $\det A = ad - bc$, $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

★ **3×3 determinant:** $\det A = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$ → Determinants and inverses of larger matrices with technology

★ **Matrix rules:** $AB \neq BA$ in general, $(AB)^{-1} = B^{-1}A^{-1}$, $AX = B \Rightarrow X = A^{-1}B$ → Multiply matrices; AB need not equal BA

★ **Eigenvalues; eigenvectors:** $\det(M-\lambda I) = 0$; $M\boldsymbol{p} = \lambda\boldsymbol{p}$ → Characteristic polynomial, eigenvalues and eigenvectors

📖 Matrix powers: $M^n = PD^nP^{-1}$

★ **P = eigenvectors as columns, D = diagonal matrix of the matching eigenvalues** → Write a 2 × 2 matrix as PDP^{-1}

Watch for List every TVM entry $(N, I\%, PV, PMT, FV, \frac{P}{Y}, \frac{C}{Y})$

Calculator use the TVM finance solver; do matrices; do complex numbers; solve equations: keystrokes at ibmathrevision.com/calculator-skills

2 Functions & models

📖 Lines: $m = \frac{y_2-y_1}{x_2-x_1}$, $y = mx+c$, $ax+by+d=0$, $y-y_1 = m(x-x_1)$

★ **Perpendicular:** $m_1 m_2 = -1$ → Finding a perpendicular line, a normal, or a perpendicular bisector

Scan to practise

📖 Quadratic: axis of symmetry: $x = -\frac{b}{2a}$

★ **Models:** $f(x) = ka^x + c$, $ke^{rx}+c$; ax^n; $a\sin(bx)+d$ (period $\frac{360°}{b}$) → Growth, decay, power and sinusoidal models in context

Watch for: Use the GDC's regression for the model named

★ **Logistic model:** $f(x) = \frac{L}{1+Ce^{-kx}}$ $(L, C, k > 0)$ → Population growth and the spread of disease

★ **Linearising:** $y = ax^b \Rightarrow \ln y = \ln a + b\ln x$; $y = ka^x \Rightarrow \ln y = \ln k + x\ln a$ → Turning a curve into a straight line to find a and b

★ **Composite; inverse:** $(f\circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$ → Composite functions in context; restrict a domain for an inverse

Watch for: f^{-1} is the inverse, f' the derivative

★ **Transformations:** $f(x)+b$ up; $f(x-a)$ right; $pf(x)$ vertical ×p; $f(qx)$ horizontal ×$\frac{1}{q}$ → Apply translations, stretches and reflections in order

Watch for Compare non-linear fits with R^2 and quote both values

Calculator do regression; find intersections and max/min: keystrokes at ibmathrevision.com/calculator-skills

3 Geometry & trigonometry

📖 3D distance ($\Delta x = x_1 - x_2$, …): $d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

📖 3D midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Scan to practise

📖 Pyramid; cone; sphere: $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, $A = \pi rl$; $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$

📖 Sine, cosine rules; area: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, $c^2 = a^2+b^2-2ab\cos C$, $A = \frac{1}{2}ab\sin C$

📖 Arc; sector, degrees: $l = \frac{\theta}{360}2\pi r$, $A = \frac{\theta}{360}\pi r^2$

📖 Arc; sector, radians: $l = r\theta$, $A = \frac{1}{2}r^2\theta$

📖 Identities: $\cos^2\theta + \sin^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

★ **Radians; bearings:** π rad $= 180°$; bearings clockwise from north → Convert between degrees and radians; three-figure bearings

★ **Perpendicular bisectors & Voronoi: cell edges on perpendicular bisectors of sites; largest empty circle at a vertex or on the boundary** → Voronoi cells; nearest-neighbour interpolation

📖 Reflection in $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

📖 Stretch ×k: horizontal, vertical; enlargement: $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$; $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$

📖 Rotation about O by θ: anticlockwise; clockwise: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$; $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

★ **Area of image:** $|\det A| \times$ area of object → The determinant as an area scale factor

★ **Composite transformation: B then A is AB** → Combine transformations

📖 Vector magnitude: $|\boldsymbol{v}| = \sqrt{v_1^2+v_2^2+v_3^2}$

📖 Scalar product; angle: $\boldsymbol{v}\cdot\boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3 = |\boldsymbol{v}||\boldsymbol{w}|\cos\theta$

📖 Vector product: $\boldsymbol{v}\times\boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$, $|\boldsymbol{v}\times\boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}|\sin\theta$

📖 Area of parallelogram: $A = |\boldsymbol{v}\times\boldsymbol{w}|$

📖 Line: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$

★ **Motion with constant velocity:** $\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v}$, speed $= |\boldsymbol{v}|$ → Positions, speeds and closest approach

★ **Unit vector; perpendicular:** $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$; $\boldsymbol{v}\cdot\boldsymbol{w} = 0$ → Unit vectors; a vector of a given length

★ **Component of $\boldsymbol{v}$ along $\boldsymbol{w}$: $\boldsymbol{v}\cdot\hat{\boldsymbol{w}}$** → Components of a vector in a given direction

★ **Graphs: sum of degrees = 2× edges; K_n has $\frac{1}{2}n(n-1)$ edges; a tree on n vertices has $n-1$ edges** → Graph theory: counting edges; can the graph exist?

★ **Planar connected graph:** $v - e + f = 2$, $e \le 3v - 6$ $(v \ge 3)$ → Recognise simple, complete and connected graphs

★ **Eulerian circuit: every vertex even; Eulerian trail: exactly two odd vertices** → Eulerian trail or circuit from vertex degrees

★ **Walks of length k from i to j:** $(A^k)_{ij}$ → Adjacency matrix questions: raise A to the power k

★ **Chinese postman: add the cheapest pairing of odd vertices, repeated once each** → Shortest route that travels every edge and returns

Watch for: Show the algorithm: list edges in order, with weights

★ **TSP: upper bound = nearest neighbour; lower bound = MST of the rest + two shortest edges from the deleted vertex** → Upper and lower bounds for the travelling salesman problem

Watch for: Upper bound: the best is the smallest found

Watch for AB with an arrow is a vector: write it as a column

Calculator do vectors; do matrices: keystrokes at ibmathrevision.com/calculator-skills

4 Statistics & probability

📖 IQR; mean: $IQR = Q_3 - Q_1$, $\bar{x} = \frac{\sum f_i x_i}{n}$

Scan to practise

★ **Outliers:** $< Q_1 - 1.5\,IQR$ or $> Q_3 + 1.5\,IQR$ → Box plots and "is this value an outlier?" questions

Watch for: Compare the actual data with the fence

📖 Probability: $P(A) = \frac{n(A)}{n(U)}$, $P(A) + P(A') = 1$

📖 Combined events: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

📖 Independent: $P(A\cap B) = P(A)P(B)$

📖 Conditional: $P(A \mid B) = \frac{P(A\cap B)}{P(B)}$

📖 Expected value: $E(X) = \sum xP(X=x)$

★ **Variance:** $\text{Var}(X) = E(X^2) - [E(X)]^2$ → Variance of a random variable

★ **Binomial pmf:** $P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$ → Fixed number of independent trials

📖 Binomial mean, variance: np, $np(1-p)$

★ **Poisson $X \sim \text{Po}(m)$:** $P(X=x) = \frac{m^x e^{-m}}{x!}$ → Random events in time or space

📖 Poisson mean, variance: $E(X) = m$, $\text{Var}(X) = m$

★ **Sum of independent Poissons:** $\text{Po}(m_1) + \text{Po}(m_2) = \text{Po}(m_1+m_2)$ → Combining two Poisson counts (two shops, two time periods)

📖 Linear combinations: $E(a_1X_1 \pm a_2X_2) = a_1E(X_1) \pm a_2E(X_2)$

📖 … independent X_i: $\text{Var}(a_1X_1 \pm a_2X_2) = a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2)$

📖 Unbiased variance estimate: $s_{n-1}^2 = \frac{n}{n-1}s_n^2$

★ **Sample mean (CLT, $n \ge 30$ or X normal):** $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ → Probabilities about a sample mean

★ **Linear combinations of independent normals are normal; for n independent $X_i \sim N(\mu, \sigma^2)$:** $\sum X_i \sim N(n\mu, n\sigma^2)$ → A linear combination of independent normals is normal

Watch for: State the final distribution before finding probabilities

📖 χ^2 statistic: $\chi^2_{\text{calc}} = \sum \frac{(f_o-f_e)^2}{f_e}$

★ **Expected frequency; df:** $\frac{\text{row}\times\text{column}}{\text{total}}$; $(r-1)(c-1)$ → Every chi-squared test for independence

★ **Goodness of fit df: classes − 1 − parameters estimated** → Chi-squared goodness-of-fit tests

★ **Reject H_0 if p-value < significance level. Type I: reject a true H_0; Type II: accept a false H_0** → Critical regions; Type I and Type II errors

Watch for: Compare the p-value with the stated level, then conclude in context

★ **Transition matrix:** $T^n s_0 = s_n$; steady state $Ts = s$, $\sum s_i = 1$ → Markov chains: states after several steps; steady state

Watch for: Each column sums to 1; after n steps use $T^n s_0$

Watch for Write H_0 and H_1 with the population parameter

Calculator do binomial and Poisson; do a normal distribution; do a t-test and a χ^2 test; find a confidence interval; do regression: keystrokes at ibmathrevision.com/calculator-skills

5 Calculus

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📖 Derivatives: $(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \frac{1}{\cos^2 x}$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$

📖 Chain, product, quotient: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, $(uv)' = uv' + vu'$, $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$

★ **Tangent; normal:** $y - f(a) = f'(a)(x-a)$; $m_n = -\frac{1}{f'(a)}$ → Any "find the equation of the tangent / normal" question

Watch for: Normal: differentiate, evaluate, take $-\frac{1}{m}$

★ **Max / min; inflexion:** $f' = 0$, $f'' < 0$ / $f'' > 0$; $f'' = 0$ with sign change → Optimisation and curve sketching

Watch for: Point of inflection: $f'' = 0$ with a sign change

★ **Related rates:** $\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt}$ → Link rates, such as how fast a tank's water level rises

📖 Integrals: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $\int \frac{1}{x}\,dx = \ln|x| + C$, $\int \sin x\,dx = -\cos x + C$

📖 More integrals: $\int \cos x\,dx = \sin x + C$, $\int \frac{1}{\cos^2 x}\,dx = \tan x + C$, $\int e^x\,dx = e^x + C$

★ **Linear inside; by inspection:** $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$; $\int g'(x)f'(g(x))\,dx = f(g(x)) + C$ → Integrate by inspection or a simple substitution

📖 Areas: $\int_a^b |y|\,dx$, $\int_c^d |x|\,dy$

📖 Volumes of revolution: $V = \int_a^b \pi y^2\,dx$, $V = \int_c^d \pi x^2\,dy$

📖 Trapezoidal rule, $h = \frac{b-a}{n}$: $\int_a^b y\,dx \approx \frac{1}{2}h\left[(y_0+y_n) + 2(y_1 + \cdots + y_{n-1})\right]$

📖 Kinematics: $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$; distance $\int_{t_1}^{t_2} |v|\,dt$

★ **Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx$ → Separate the variables; use the initial condition

Watch for: One $+C$, used at once with the initial condition

📖 Euler's method: $y_{n+1} = y_n + h\,f(x_n, y_n)$, $x_{n+1} = x_n + h$

📖 Euler, coupled systems: $x_{n+1} = x_n + h\,f_1(x_n, y_n, t_n)$, $y_{n+1} = y_n + h\,f_2(x_n, y_n, t_n)$, $t_{n+1} = t_n + h$

📖 Coupled linear system $\dot{\boldsymbol{x}} = M\boldsymbol{x}$: $\boldsymbol{x} = Ae^{\lambda_1 t}\boldsymbol{p}_1 + Be^{\lambda_2 t}\boldsymbol{p}_2$

★ **Phase portraits: real λ both < 0 stable node, both > 0 unstable, opposite signs saddle; complex: $\text{Re}\,\lambda < 0$ spiral in, > 0 out, $= 0$ closed loops** → Classify the equilibrium from the eigenvalues

Watch for: Equilibrium: $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$

★ **2nd order $\ddot{x} + a\dot{x} + bx = 0$:** put $y = \dot{x}$: $\dot{x} = y$, $\dot{y} = -bx - ay$ → Second-order equations, such as a spring, as coupled systems

Watch for Show the first Euler step with numbers

Calculator find derivatives and integrals; find intersections and max/min; make a table of values: keystrokes at ibmathrevision.com/calculator-skills

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