FORMULAS YOU MUST MEMORISE

IB Mathematics: Analysis & Approaches SL

Not in the IB formula booklet: learn these by heart.

★ Not given in the exam: learn every formula on this page. The full sheet and the booklet have the rest.

Prior learning

★ **Pythagoras:** $a^2 + b^2 = c^2$
When: Any right-angled triangle, in 2D or 3D

★ **Right-angled trig:** $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$
When: Sides and angles in right-angled triangles, bearings, elevation

★ **Laws of indices:** $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
When: Simplifying powers and roots without a calculator

★ **Standard form:** $a \times 10^k$, $1 \le a < 10,\ k \in \mathbb{Z}$
When: Converting numbers to and from standard form

5 Calculus

★ **Linear inside:** $\frac{d}{dx}e^{kx} = ke^{kx}$, $\frac{d}{dx}\sin kx = k\cos kx$, $\frac{d}{dx}\ln(kx) = \frac{1}{x}$
When: Composite functions such as e^{2x} and $\sin(4x+1)$

★ **Tangent; normal gradient at $x = a$:** $y - f(a) = f'(a)(x-a)$; $m_{\text{normal}} = -\frac{1}{f'(a)}$
When: Any "find the equation of the tangent / normal" question

★ **Increasing / decreasing:** $f'(x) > 0$; $f'(x) < 0$
When: Where a function is increasing or decreasing

★ **Stationary points:** $f'(x) = 0$: $f'' < 0$ **max**, $f'' > 0$ **min**
When: Optimisation and curve sketching

★ **Concave up $f'' > 0$; point of inflexion: $f'' = 0$ and f'' changes sign**
When: Concave up or down; points of inflexion

★ **Linear inside:** $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$
When: Integrals like $(3x-1)^4$, e^{2x+1} or $\cos(5x)$

★ **Reverse chain rule:** $\int f'\big(g(x)\big)g'(x)\,dx = f\big(g(x)\big) + C$
When: Integrate $f'(x)[f(x)]^n$, $\frac{f'(x)}{f(x)}$ or $f'(x)e^{f}(x)$ by inspection

★ **Definite integral:** $\int_a^b f'(x)\,dx = f(b) - f(a)$
When: From f' to f: integrate, add $+C$, use the given point

★ **Area between two curves:** $A = \int_a^b |f(x) - g(x)|\,dx$
When: Shaded-region questions

★ **Displacement $t_1 \to t_2$:** $\int_{t_1}^{t_2} v(t)\,dt$
When: Displacement or position from velocity, with a boundary condition

1 Number & algebra

★ **Special values:** $\log_a 1 = 0$, $\log_a a = 1$, $\ln e = 1$, $e^{\ln x} = x$
When: Simplifying logarithm expressions exactly

★ **General term:** $T_{r+1} = \binom{n}{r}a^{n-r}b^r$
When: Find a specified term, coefficient or constant term

2 Functions

★ **Parallel; perpendicular lines:** $m_1 = m_2$; $m_1 m_2 = -1$
When: Equation of a parallel or perpendicular line through a point

★ **Roots: $\Delta > 0$ two distinct real, $\Delta = 0$ one repeated, $\Delta < 0$ no real roots**
When: Count real roots; quadratic inequalities with a sketch

★ **Vertex form, vertex (h, k); intercept form:** $y = a(x-h)^2 + k$; $y = a(x-p)(x-q)$
When: Finding a turning point, a range, or transformations of $y = x^2$

★ **Composite; inverse:** $(f \circ g)(x) = f\big(g(x)\big)$; $f\big(f^{-1}(x)\big) = x$
When: Finding $f^{-1}(x)$: swap x and y, then make y the subject

★ **Graph of f^{-1}: reflect $y = f(x)$ in $y = x$; domain of f^{-1} = range of f**
When: Sketch $y = f^{-1}(x)$ and state its domain and range

★ **Translations: $f(x) + b$ up b; $f(x-a)$ right a**
When: Sketch $y = f(x)$+ a and $y = f(x+a)$, tracking key points

★ **Stretches: $pf(x)$ vertical ×p; $f(qx)$ horizontal ×$\frac{1}{q}$**
When: Vertical and horizontal stretches: state the scale factor

★ **Reflections: $-f(x)$ in the x-axis; $f(-x)$ in the y-axis**
When: Reflect graphs in the axes; apply transformations in order

★ **$y = \frac{ax+b}{cx+d}$ asymptotes:** $x = -\frac{d}{c}$, $y = \frac{a}{c}$
When: Sketching reciprocal-type graphs; domains and ranges

★ **Exponential ↔ log:** $y = a^x \iff x = \log_a y$; e^x **and** $\ln x$ **are inverses**
When: Growth and decay problems combining exponential and log equations

3 Geometry & trigonometry

★ **Ambiguous case: given a, b, A with A acute, two triangles if $b\sin A < a < b$**
When: When two triangles fit: find both angles, decide which are valid

★ **Radians:** π rad $= 180°$
When: Convert between degrees and radians

★ **Exact values at $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$:**
sin : $0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$;
cos : reverse order;
tan : $0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$, undefined
When: Every non-calculator trig question

★ **Symmetry:** $\sin(\pi - \theta) = \sin\theta$, $\cos(\pi - \theta) = -\cos\theta$, $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$
When: Exact values for any angle from the unit circle

★ **$y = a\sin\big(b(x-c)\big) + d$, $b > 0$:** amplitude $|a|$, period $\frac{2\pi}{b}$, principal axis $y = d$
When: Matching a graph to its equation, and modelling

★ **$\sin x = k$: $x = \arcsin k$ or $\pi - \arcsin k$, then add $2\pi n$; keep solutions in the interval**
When: Solving trig equations over an interval: the calculator gives one answer

4 Statistics & probability

★ **Outliers:** $x < Q_1 - 1.5\,IQR$ or $x > Q_3 + 1.5\,IQR$
When: Box plots and "is this value an outlier?" questions

★ **Variance (s.d. σ):** $\sigma^2 = \frac{\sum f(x-\mu)^2}{n} = \frac{\sum fx^2}{n} - \mu^2$
When: Spread of data; check with technology

★ **Data ×a then +b: mean $\to a\bar{x} + b$, s.d. $\to |a|\sigma$**
When: Questions that change units or scale marks

★ **Regression: use y on x to predict y, x on y to predict x; interpolate, don't extrapolate**
When: Choose the y-on-x or x-on-y line for a prediction

★ **Valid distribution; fair game:** $\sum P(X = x) = 1$; $E(\text{gain}) = 0$
When: Find an unknown probability; decide whether a game is fair

★ **Binomial $X \sim B(n, p)$:** $P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$
When: Fixed number of independent trials

★ **Normal: about 68%, 95%, 99.7% of values lie within 1, 2, 3 s.d. of μ**
When: Estimate probabilities from the symmetry of the normal curve

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