IB Math Revision
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IB Mathematics: Analysis & Approaches SL

One-page formula sheet · booklet formulas + ★ the ones you must learn

★ not in the IB formula booklet — learn it. Everything else is printed in the booklet (its notation may differ slightly).

Prior learning

Area: parallelogram, triangle, trapezoid: $A = bh$, $A = \frac{1}{2}bh$, $A = \frac{1}{2}(a+b)h$

Circle: $A = \pi r^2$, $C = 2\pi r$

Volume: cuboid, cylinder, prism: $V = lwh$, $V = \pi r^2 h$, $V = Ah$

Curved surface of a cylinder: $A = 2\pi rh$

Distance; midpoint: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

★ Pythagoras: $a^2 + b^2 = c^2$

★ Right-angled trig: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

★ Laws of indices: $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

★ Standard form: $a \times 10^k$, $1 \le a < 10$, $k \in \mathbb{Z}$

1 Number & algebra

Arithmetic sequence: $u_n = u_1 + (n-1)d$

Arithmetic series: $S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right) = \frac{n}{2}(u_1 + u_n)$

Geometric sequence: $u_n = u_1 r^{n-1}$

Geometric series, $r \ne 1$: $S_n = \frac{u_1(r^n-1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$

Sum to infinity, $|r| < 1$: $S_\infty = \frac{u_1}{1-r}$

Compound interest (k times a year, n years, r%): $FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$

Exponents & logs: $a^x = b \iff x = \log_a b$, $a^x = e^{x\ln a}$, $\log_a a^x = x = a^{\log_a x}$

Laws of logs: $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^m = m\log_a x$

Change of base: $\log_b a = \frac{\log_c a}{\log_c b}$

★ Special values: $\log_a 1 = 0$, $\log_a a = 1$, $\ln e = 1$, $e^{\ln x} = x$

Binomial theorem, $n \in \mathbb{N}$: $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$

Binomial coefficient: $\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$

★ General term: $T_{r+1} = \binom{n}{r}a^{n-r}b^r$

Deductive proof: work from one side to the other, one justified step per line.

2 Functions

Gradient; forms of a line: $m = \frac{y_2-y_1}{x_2-x_1}$, $y = mx + c$, $ax + by + d = 0$, $y - y_1 = m(x - x_1)$

★ Parallel; perpendicular lines: $m_1 = m_2$; $m_1 m_2 = -1$

Quadratic: axis of symmetry: $x = -\frac{b}{2a}$

Quadratic formula; discriminant: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $\Delta = b^2 - 4ac$

★ Roots: $\Delta > 0$ two distinct real, $\Delta = 0$ one repeated, $\Delta < 0$ no real roots

★ Vertex form, vertex (h,k); intercept form: $y = a(x-h)^2 + k$; $y = a(x-p)(x-q)$

★ Composite; inverse: $(f \circ g)(x) = f\big(g(x)\big)$; $f\big(f^{-1}(x)\big) = x$

★ Graph of f^{-1}: reflect $y = f(x)$ in $y = x$; domain of f^{-1} = range of f

★ Translations: $f(x) + b$ up b; $f(x-a)$ right a

★ Stretches: $pf(x)$ vertical $\times p$; $f(qx)$ horizontal $\times\frac{1}{q}$

★ Reflections: $-f(x)$ in the x-axis; $f(-x)$ in the y-axis

★ $y = \frac{ax+b}{cx+d}$ asymptotes: $x = -\frac{d}{c}$, $y = \frac{a}{c}$

★ Exponential ↔ log: $y = a^x \iff x = \log_a y$; e^x and $\ln x$ are inverses

3 Geometry & trigonometry

3D distance: $d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$

3D midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Pyramid; cone: $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, curved $A = \pi rl$

Sphere: $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

Area of a triangle: $A = \frac{1}{2}ab\sin C$

★ Ambiguous case: given a, b, A with A acute, two triangles if $b\sin A < a < b$

Arc; sector (θ in radians): $l = r\theta$, $A = \frac{1}{2}r^2\theta$

★ Radians: π rad $= 180°$

Identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos^2\theta + \sin^2\theta = 1$

Double angle: $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

★ Exact values at $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$: $\sin:\ 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$; $\cos:$ reverse order; $\tan:\ 0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$, undefined

★ Symmetry: $\sin(\pi-\theta) = \sin\theta$, $\cos(\pi-\theta) = -\cos\theta$, $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$

★ $y = a\sin\big(b(x-c)\big) + d$, $b > 0$: amplitude $|a|$, period $\frac{2\pi}{b}$, principal axis $y = d$

★ $\sin x = k$: $x = \arcsin k$ or $\pi - \arcsin k$, then add $2\pi n$; keep solutions in the interval

4 Statistics & probability

Interquartile range: $IQR = Q_3 - Q_1$

Mean ($n = \sum f_i$): $\bar{x} = \frac{\sum f_i x_i}{n}$

★ Outliers: $x < Q_1 - 1.5\,IQR$ or $x > Q_3 + 1.5\,IQR$

★ Variance (s.d. σ): $\sigma^2 = \frac{\sum f(x-\mu)^2}{n} = \frac{\sum fx^2}{n} - \mu^2$

★ Data $\times a$ then $+b$: mean $\to a\bar{x} + b$, s.d. $\to |a|\sigma$

★ Regression: use y on x to predict y, x on y to predict x; interpolate, don't extrapolate

Probability: $P(A) = \frac{n(A)}{n(U)}$, $P(A) + P(A') = 1$

Combined events: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

Mutually exclusive: $P(A\cup B) = P(A) + P(B)$

Independent: $P(A\cap B) = P(A)P(B)$

Conditional: $P(A \mid B) = \frac{P(A\cap B)}{P(B)}$

Expected value: $E(X) = \sum x\,P(X = x)$

★ Valid distribution; fair game: $\sum P(X=x) = 1$; $E(\text{gain}) = 0$

★ Binomial $X \sim B(n,p)$: $P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$

Binomial mean, variance: $E(X) = np$, $\mathrm{Var}(X) = np(1-p)$

Standardised normal: $z = \frac{x-\mu}{\sigma}$

★ Normal: about 68%, 95%, 99.7% of values lie within 1, 2, 3 s.d. of μ

5 Calculus

Power rule: $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

Standard derivatives: $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$

Chain rule: $y = g(u)$, $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

Product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

★ Linear inside: $\frac{d}{dx}e^{kx} = ke^{kx}$, $\frac{d}{dx}\sin kx = k\cos kx$, $\frac{d}{dx}\ln(kx) = \frac{1}{x}$

★ Tangent; normal gradient at $x = a$: $y - f(a) = f'(a)(x-a)$; $m_{\text{normal}} = -\frac{1}{f'(a)}$

★ Increasing / decreasing: $f'(x) > 0$; $f'(x) < 0$

★ Stationary points: $f'(x) = 0$: $f'' < 0$ max, $f'' > 0$ min

★ Concave up $f'' > 0$; point of inflexion: $f'' = 0$ and f'' changes sign

Integral of x^n, $n \ne -1$: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Standard integrals: $\int \frac{1}{x}\,dx = \ln|x| + C$, $\int \sin x\,dx = -\cos x + C$, $\int \cos x\,dx = \sin x + C$, $\int e^x\,dx = e^x + C$

★ Linear inside: $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$

★ Reverse chain rule: $\int f'\big(g(x)\big)g'(x)\,dx = f\big(g(x)\big) + C$

★ Definite integral: $\int_a^b f'(x)\,dx = f(b) - f(a)$

Area, curve and x-axis: $A = \int_a^b |y|\,dx$

★ Area between two curves: $A = \int_a^b |f(x) - g(x)|\,dx$

Kinematics: $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Distance travelled $t_1 \to t_2$: $\int_{t_1}^{t_2} |v(t)|\,dt$

★ Displacement $t_1 \to t_2$: $\int_{t_1}^{t_2} v(t)\,dt$