FORMULAS YOU MUST MEMORISE

# IB Mathematics: Analysis & Approaches HL

Not in the IB formula booklet: learn these by heart.



★ Not given in the exam: learn every formula on this page. The full sheet and the booklet have the rest.

## Prior learning

★ **Indices:** $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
**When:** Simplify negative and fractional powers; solve exponential equations

## 1 Number & algebra

★ $(a+bx)^n = a^n\left(1+\frac{b}{a}x\right)^n$, **valid for** $|x| < \left|\frac{a}{b}\right|$
**When:** Expanding $(a+bx)^n$ for negative or fractional $n$

★ **Partial fractions:** $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$
**When:** Integration and series expansions of rational functions

★ **Modulus, argument, conjugate:** $r = |z| = \sqrt{a^2+b^2}$, $\tan\theta = \frac{b}{a}$, $z^* = a - bi$, $zz^* = |z|^2$
**When:** Plot on an Argand diagram; find the modulus and argument

★ **Products:** $|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$, $\arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$
**When:** Multiply and divide in polar form

★ **$n$th roots of $re^{i\theta}$:** $r^{1/n}e^{i(\theta+2\pi k)/n}$, $k = 0, 1, \ldots, n-1$
**When:** Solving $z^n = w$: de Moivre gives powers, not the full set of roots

★ **For $|z| = 1$:** $z^n + z^{-n} = 2\cos n\theta$, $z^n - z^{-n} = 2i\sin n\theta$
**When:** Writing $\cos^n\theta$ or $\sin^n\theta$ in terms of multiple angles

★ **Roots of unity sum to 0; non-real roots of real polynomials come in conjugate pairs**
**When:** Find remaining roots: complex roots come in conjugate pairs

★ **Induction: prove $P(1)$; assume $P(k)$; prove $P(k+1)$; conclude for all $n \in \mathbb{Z}^+$**
**When:** Summation formulas and divisibility statements

★ **Contradiction: assume the negation, reach a contradiction. Counterexample disproves**
**When:** Irrationality of $\sqrt{2}$; disprove with one counterexample

## 2 Functions

★ **Perpendicular:** $m_1 m_2 = -1$
**When:** Finding a perpendicular line, a normal, or a perpendicular bisector

★ **$\Delta > 0$: two real roots; $\Delta = 0$: repeated root; $\Delta < 0$: complex conjugate roots**
**When:** Count real roots and find unknown constants

★ **Factor & remainder theorems:** $p(a) = 0 \iff (x-a)$ **is a factor; remainder of** $p(x) \div (x-a)$ **is** $p(a)$
**When:** Finding unknown coefficients and factorising cubics

★ **Odd; even:** $f(-x) = -f(x)$; $f(-x) = f(x)$
**When:** Identify even and odd functions

★ **Composite; inverse:** $(f \circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$
**When:** Finding $f^{-1}(x)$: swap $x$ and $y$, then make $y$ the subject

★ **Transformations:** $f(x)+b$ **up** $b$; $f(x-a)$ **right** $a$; $pf(x)$ **vertical** $\times p$; $f(qx)$ **horizontal** $\times\frac{1}{q}$; $-f(x)$, $f(-x)$ **reflect in** $x$-, $y$-**axis**
**When:** Sketch translations and stretches, tracking key points

★ **Graphs:** $|f(x)|$ **reflects negative parts up;** $f(|x|)$ **mirrors** $x \geq 0$; $\frac{1}{f(x)}$ **has asymptotes at zeros of** $f$
**When:** Sketch $|f(x)|$, $f(|x|)$, $\frac{1}{f(x)}$ and $[f(x)]^2$, tracking roots and asymptotes

★ $y = \frac{ax+b}{cx+d}$ **asymptotes:** $x = -\frac{d}{c}$, $y = \frac{a}{c}$
**When:** Sketching reciprocal-type graphs; domains and ranges

## 3 Geometry & trigonometry

★ **Exact values** $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$: $\sin: 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$; $\cos$: **reverse**; $\tan: 0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$, **undef.**
**When:** Every non-calculator trig question

★ **Ranges:** $\arcsin x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\arccos x \in [0, \pi]$, $\arctan x \in (-\frac{\pi}{2}, \frac{\pi}{2})$
**When:** Inverse trig functions with their domains and ranges

★ $a\sin(b(x-c))+d$, $b > 0$: **amplitude** $|a|$, **period** $\frac{2\pi}{b}$
**When:** Matching a graph to its equation, and modelling

★ **Perpendicular; unit vector;** $\overrightarrow{AB}$: $\boldsymbol{v}\cdot\boldsymbol{w} = 0$; $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$; $\boldsymbol{b}-\boldsymbol{a}$
**When:** Test whether two vectors are perpendicular; find unit vectors

★ **Area of triangle:** $\frac{1}{2}|\boldsymbol{v}\times\boldsymbol{w}|$
**When:** 3D triangle areas (the booklet gives the parallelogram)

★ **Angle line–plane; plane–plane:** $\sin\theta = \frac{|\boldsymbol{b}\cdot\boldsymbol{n}|}{|\boldsymbol{b}||\boldsymbol{n}|}$; $\cos\theta = \frac{|\boldsymbol{n}_1\cdot\boldsymbol{n}_2|}{|\boldsymbol{n}_1||\boldsymbol{n}_2|}$
**When:** Angles between lines and planes

★ **Normal to plane through $\boldsymbol{a}$ with directions $\boldsymbol{b}, \boldsymbol{c}$:** $\boldsymbol{n} = \boldsymbol{b}\times\boldsymbol{c}$
**When:** Normal vector found with the vector product

★ **Line of intersection of two planes: direction** $\boldsymbol{n}_1 \times \boldsymbol{n}_2$
**When:** Line of intersection of two planes

## 4 Statistics & probability

★ **Outliers:** $< Q_1 - 1.5\,IQR$ **or** $> Q_3 + 1.5\,IQR$
**When:** Box plots and "is this value an outlier?" questions

★ **Variance:** $\sigma^2 = \frac{\sum f(x-\mu)^2}{n} = \frac{\sum fx^2}{n} - \mu^2$
**When:** Variance and standard deviation as measures of spread

★ **Binomial pmf:** $P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$
**When:** Fixed number of independent trials

★ **pdf; median** $m$: $\int f(x)\,dx = 1$; $\int_{-\infty}^{m} f(x)\,dx = \frac{1}{2}$; **mode: max of** $f$
**When:** Continuous random variable questions

★ **Regression:** $y$ **on** $x$ **to predict** $y$, $x$ **on** $y$ **for** $x$; **interpolate only**
**When:** Choose the right regression line for a prediction

## 5 Calculus

★ **Implicit:** $\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$, $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$
**When:** Gradients of curves you cannot write as $y = \ldots$

★ **Related rates:** $\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt}$
**When:** Link rates of change, such as a growing balloon

★ **Tangent; normal:** $y - f(a) = f'(a)(x-a)$; $m_n = -\frac{1}{f'(a)}$
**When:** Any "find the equation of the tangent / normal" question

★ **Max / min; inflexion:** $f' = 0$ **and** $f'' < 0$ / $f'' > 0$; $f'' = 0$ **with sign change**
**When:** Optimisation and curve sketching

★ **L'Hôpital, for $\frac{0}{0}$ or $\frac{\infty}{\infty}$:** $\lim\frac{f(x)}{g(x)} = \lim\frac{f'(x)}{g'(x)}$
**When:** Limits that give $\frac{0}{0}$

★ **Learn:** $\int\sec^2 x\,dx = \tan x + C$, $\int\frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$, $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$
**When:** Any fraction whose top is (a multiple of) the derivative of the bottom

★ **$\sin^2 x$, $\cos^2 x$:** $\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$
**When:** Integrating even powers of sin and cos

★ **Substitution:** $\int f(g(x))g'(x)\,dx = \int f(u)\,du$; **change the limits too**
**When:** Integrate by substitution, changing the limits

★ **Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int\frac{dy}{g(y)} = \int f(x)\,dx$
**When:** Solve by separating the variables, with an initial condition

★ **Homogeneous** $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$: $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$
**When:** First-order equations where every term has the same degree

★ **Then:** $ye^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx$
**When:** Solve linear differential equations with an integrating factor


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