★ not in the IB formula booklet — learn it. Everything else is printed in the booklet (its notation may differ slightly).

Prior learning

Areas: parallelogram bh, triangle $\frac{1}{2}bh$, trapezoid $\frac{1}{2}(a+b)h$, circle πr^2

Circumference; cylinder: $C = 2\pi r$; $V = \pi r^2 h$, curved $A = 2\pi rh$

Cuboid; prism: $V = lwh$; $V = Ah$

Distance; midpoint: $d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

★ Indices: $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

1 Number & algebra

Arithmetic: $u_n = u_1 + (n-1)d$, $S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right) = \frac{n}{2}(u_1 + u_n)$

Geometric: $u_n = u_1 r^{n-1}$, $S_n = \frac{u_1(1-r^n)}{1-r}$, $S_\infty = \frac{u_1}{1-r}$ $(|r| < 1)$

Compound interest: $FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$

Exponents & logs: $a^x = b \iff x = \log_a b$, $a^x = e^{x \ln a}$, $\log_a a^x = x = a^{\log_a x}$

Log laws: $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^m = m \log_a x$

Change of base: $\log_b a = \frac{\log_c a}{\log_c b}$

Binomial, $n \in \mathbb{N}$: $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$, $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Permutations: ${}^nP_r = \frac{n!}{(n-r)!}$

Binomial, $n \in \mathbb{Q}$, $|x| < 1$: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$

★ $(a+bx)^n = a^n\left(1 + \frac{b}{a}x\right)^n$, valid for $|x| < \left|\frac{a}{b}\right|$

★ Partial fractions: $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$

Complex numbers: $z = a + bi = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\operatorname{cis}\theta$

★ Modulus, argument, conjugate: $r = |z| = \sqrt{a^2+b^2}$, $\tan\theta = \frac{b}{a}$, $z^* = a - bi$, $zz^* = |z|^2$

★ Products: $|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$, $\arg \frac{z_1}{z_2} = \arg z_1 - \arg z_2$

De Moivre: $\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta}$

★ nth roots of $re^{i\theta}$: $r^{1/n}e^{i(\theta+2\pi k)/n}$, $k = 0, 1, \ldots, n-1$

★ For $|z| = 1$: $z^n + z^{-n} = 2\cos n\theta$, $z^n - z^{-n} = 2i\sin n\theta$

★ Roots of unity sum to 0; non-real roots of real polynomials come in conjugate pairs

★ Induction: prove $P(1)$; assume $P(k)$; prove $P(k+1)$; conclude for all $n \in \mathbb{Z}^+$

★ Contradiction: assume the negation, reach a contradiction. Counterexample disproves

2 Functions

Lines: $m = \frac{y_2-y_1}{x_2-x_1}$, $y = mx + c$, $ax + by + d = 0$, $y - y_1 = m(x - x_1)$

★ Perpendicular: $m_1 m_2 = -1$

Quadratics: $x = -\frac{b}{2a}$, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $\Delta = b^2 - 4ac$

★ $\Delta > 0$: two real roots; $\Delta = 0$: repeated root; $\Delta < 0$: complex conjugate roots

Sum, product of roots of $\sum_0^n a_r x^r$: $-\frac{a_{n-1}}{a_n}$; $\frac{(-1)^n a_0}{a_n}$

★ Factor & remainder theorems: $p(a) = 0 \iff (x-a)$ is a factor; remainder of $p(x) \div (x-a)$ is $p(a)$

★ Odd; even: $f(-x) = -f(x)$; $f(-x) = f(x)$

★ Composite; inverse: $(f \circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$

★ Transformations: $f(x) + b$ up b; $f(x-a)$ right a; $pf(x)$ vertical ×p; $f(qx)$ horizontal ×$\frac{1}{q}$; $-f(x)$, $f(-x)$ reflect in x-, y-axis

★ Graphs: $|f(x)|$ reflects negative parts up; $f(|x|)$ mirrors $x \geq 0$; $\frac{1}{f(x)}$ has asymptotes at zeros of f

★ $y = \frac{ax+b}{cx+d}$ asymptotes: $x = -\frac{d}{c}$, $y = \frac{a}{c}$

3 Geometry & trigonometry

3D distance ($\Delta x = x_1 - x_2, \ldots$): $d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

3D midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Pyramid; cone; sphere: $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, $A = \pi rl$; $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$

Sine, cosine rules; area: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, $c^2 = a^2 + b^2 - 2ab\cos C$, $A = \frac{1}{2}ab\sin C$

Arc; sector (radians): $l = r\theta$, $A = \frac{1}{2}r^2\theta$

Identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos^2\theta + \sin^2\theta = 1$

Reciprocal ratios: $\sec\theta = \frac{1}{\cos\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

Pythagorean: $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Compound angles: $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$, $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

Compound tan: $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Double angle: $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$, $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

★ Exact values $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$: $\sin: 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$; $\cos:$ reverse; $\tan: 0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$, undef.

★ Ranges: $\arcsin x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\arccos x \in [0, \pi]$, $\arctan x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

★ $a\sin(b(x-c)) + d$, $b > 0$: amplitude $|a|$, period $\frac{2\pi}{b}$

Vector magnitude: $|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Scalar product: $\boldsymbol{v} \cdot \boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3 = |\boldsymbol{v}||\boldsymbol{w}|\cos\theta$

★ Perpendicular; unit vector; $\overrightarrow{AB}$: $\boldsymbol{v} \cdot \boldsymbol{w} = 0$; $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$; $\boldsymbol{b} - \boldsymbol{a}$

Line: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$; $x = x_0 + \lambda l, \ldots$; $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$

Vector product: $\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$, $|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}|\sin\theta$

Area of parallelogram: $A = |\boldsymbol{v} \times \boldsymbol{w}|$

★ Area of triangle: $\frac{1}{2}|\boldsymbol{v} \times \boldsymbol{w}|$

Plane: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$, $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$, $ax + by + cz = d$

★ Angle line–plane; plane–plane: $\sin\theta = \frac{|\boldsymbol{b}\cdot\boldsymbol{n}|}{|\boldsymbol{b}||\boldsymbol{n}|}$; $\cos\theta = \frac{|\boldsymbol{n}_1\cdot\boldsymbol{n}_2|}{|\boldsymbol{n}_1||\boldsymbol{n}_2|}$

★ Normal to plane through $\boldsymbol{a}$ with directions $\boldsymbol{b}, \boldsymbol{c}$: $\boldsymbol{n} = \boldsymbol{b} \times \boldsymbol{c}$

★ Line of intersection of two planes: direction $\boldsymbol{n}_1 \times \boldsymbol{n}_2$

4 Statistics & probability

IQR; mean: $IQR = Q_3 - Q_1$, $\bar{x} = \frac{\sum f_i x_i}{n}$

★ Outliers: $< Q_1 - 1.5\,IQR$ or $> Q_3 + 1.5\,IQR$

★ Variance: $\sigma^2 = \frac{\sum f(x-\mu)^2}{n} = \frac{\sum fx^2}{n} - \mu^2$

Probability: $P(A) = \frac{n(A)}{n(U)}$, $P(A) + P(A') = 1$

Combined events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Independent: $P(A \cap B) = P(A)P(B)$

Conditional: $P(A \mid B) = \frac{P(A\cap B)}{P(B)}$

Bayes: $P(B \mid A) = \frac{P(B)P(A|B)}{P(B)P(A|B) + P(B')P(A|B')}$

Bayes, partition B_1, B_2, B_3: $P(B_i \mid A) = \frac{P(B_i)P(A|B_i)}{\sum_j P(B_j)P(A|B_j)}$

Discrete X: $E(X) = \sum xP(X=x)$, $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$

★ Binomial pmf: $P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$

Binomial mean, variance: np, $np(1-p)$

Standardised normal: $z = \frac{x-\mu}{\sigma}$

Continuous X: $E(X) = \int x\,f(x)\,dx$, $\mathrm{Var}(X) = \int x^2 f(x)\,dx - \mu^2$

★ pdf; median m: $\int f(x)\,dx = 1$; $\int_{-\infty}^{m} f(x)\,dx = \frac{1}{2}$; mode: max of f

Linear transformation: $E(aX + b) = aE(X) + b$, $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$

★ Regression: y on x to predict y, x on y for x; interpolate only

5 Calculus

First principles: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Basic derivatives: $(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$

Trig derivatives: $(\tan x)' = \sec^2 x$, $(\sec x)' = \sec x\tan x$, $(\operatorname{cosec} x)' = -\operatorname{cosec} x\cot x$, $(\cot x)' = -\operatorname{cosec}^2 x$

More derivatives: $(a^x)' = a^x \ln a$, $(\log_a x)' = \frac{1}{x\ln a}$

Inverse trig: $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$, $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$, $(\arctan x)' = \frac{1}{1+x^2}$

Chain, product, quotient: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, $(uv)' = uv' + vu'$, $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$

★ Implicit: $\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$, $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$

★ Related rates: $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

★ Tangent; normal: $y - f(a) = f'(a)(x-a)$; $m_n = -\frac{1}{f'(a)}$

★ Max / min; inflexion: $f' = 0$ and $f'' < 0$ / $f'' > 0$; $f'' = 0$ with sign change

★ L'Hôpital, for $\frac{0}{0}$ or $\frac{\infty}{\infty}$: $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$

Standard integrals: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $\int \frac{1}{x}\,dx = \ln|x| + C$, $\int \sin x\,dx = -\cos x + C$, $\int \cos x\,dx = \sin x + C$, $\int e^x\,dx = e^x + C$

More integrals: $\int a^x\,dx = \frac{a^x}{\ln a} + C$, $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$, $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

★ Learn: $\int \sec^2 x\,dx = \tan x + C$, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$, $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$

★ $\sin^2 x$, $\cos^2 x$: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

By parts: $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

★ Substitution: $\int f(g(x))g'(x)\,dx = \int f(u)\,du$; change the limits too

Areas: $\int_a^b |y|\,dx$, $\int_c^d |x|\,dy$

Volumes of revolution: $V = \int_a^b \pi y^2\,dx$, $V = \int_c^d \pi x^2\,dy$

Kinematics: $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$; distance $\int |v|\,dt$

Euler's method: $y_{n+1} = y_n + h\,f(x_n, y_n)$, $x_{n+1} = x_n + h$

★ Separable: $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx$

★ Homogeneous $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$: $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Integrating factor for $y' + P(x)y = Q(x)$: $e^{\int P(x)\,dx}$

★ Then: $y e^{\int P\,dx} = \int Q e^{\int P\,dx}\,dx$

Maclaurin: $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$

Series: $e^x = 1 + x + \frac{x^2}{2!} + \cdots$, $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

sin, cos: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$, $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

arctan: $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$