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FORMULA POSTER

# IB Mathematics: Analysis & Approaches HL

2021 syllabus · v2 Oct 2026

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formula-sheet-hlaa
Interactive version

★ Learn it: not in the IB formula booklet  📖 In the booklet

## Prior learning

📖 Areas: parallelogram $bh$, triangle $\frac{1}{2}bh$, trapezoid $\frac{1}{2}(a+b)h$, circle $\pi r^2$

📖 Circumference; cylinder: $C = 2\pi r$; $V = \pi r^2 h$, curved $A = 2\pi rh$

📖 Cuboid; prism: $V = lwh$; $V = Ah$

Scan to practise

📖 Distance; midpoint: $d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

★ **Indices:** $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ → Simplify negative and fractional powers; solve exponential equations

## 1 Number & algebra

📖 Arithmetic: $u_n = u_1 + (n-1)d$, $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

📖 Geometric: $u_n = u_1 r^{n-1}$, $S_n = \frac{u_1(1-r^n)}{1-r}$, $S_\infty = \frac{u_1}{1-r}$ $(|r|<1)$

📖 Compound interest: $FV = PV\left(1+\frac{r}{100k}\right)^{kn}$

📖 Exponents & logs: $a^x = b \iff x = \log_a b$, $a^x = e^{x\ln a}$, $\log_a a^x = x = a^{\log_a x}$

Scan to practise

📖 Log laws: $\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^m = m\log_a x$

📖 Change of base: $\log_b a = \frac{\log_c a}{\log_c b}$

📖 Binomial, $n \in \mathbb{N}$: $(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$, $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

📖 Permutations: ${}^nP_r = \frac{n!}{(n-r)!}$

📖 Binomial, $n \in \mathbb{Q}$, $|x|<1$: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$

★ $(a+bx)^n = a^n\left(1+\frac{b}{a}x\right)^n$, **valid for** $|x| < \left|\frac{a}{b}\right|$ → Expanding $(a+bx)^n$ for negative or fractional $n$

★ **Partial fractions:** $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$ → Integration and series expansions of rational functions

📖 Complex numbers: $z = a+bi = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\operatorname{cis}\theta$

★ **Modulus, argument, conjugate:** $r = |z| = \sqrt{a^2+b^2}$, $\tan\theta = \frac{b}{a}$, $z^* = a-bi$, $zz^* = |z|^2$ → Plot on an Argand diagram; find the modulus and argument

★ **Products:** $|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$, $\arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$ → Multiply and divide in polar form

📖 De Moivre: $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta}$

★ **$n$th roots of $re^{i\theta}$:** $r^{1/n}e^{i(\theta+2\pi k)/n}$, $k = 0, 1, \ldots, n-1$ → Solving $z^n = w$: de Moivre gives powers, not the full set of roots
Watch for: Find all the roots, not just one

★ **For $|z|=1$:** $z^n + z^{-n} = 2\cos n\theta$, $z^n - z^{-n} = 2i\sin n\theta$ → Writing $\cos^n\theta$ or $\sin^n\theta$ in terms of multiple angles

★ **Roots of unity sum to 0; non-real roots of real polynomials come in conjugate pairs** → Find remaining roots: complex roots come in conjugate pairs
Watch for: A real polynomial with root $a+bi$ also has root $a-bi$

★ **Induction: prove $P(1)$; assume $P(k)$; prove $P(k+1)$; conclude for all $n \in \mathbb{Z}^+$** → Summation formulas and divisibility statements
Watch for: Four parts in words, ending with the conclusion

★ **Contradiction: assume the negation, reach a contradiction. Counterexample disproves** → Irrationality of $\sqrt{2}$; disprove with one counterexample

**Watch for** Four parts in words, ending with the conclusion

**Calculator** do complex numbers; find sums of sequences: keystrokes at ibmathrevision.com/calculator-skills

## 2 Functions

📖 Lines: $m = \frac{y_2-y_1}{x_2-x_1}$, $y = mx + c$, $ax + by + d = 0$, $y - y_1 = m(x - x_1)$

★ **Perpendicular:** $m_1 m_2 = -1$ → Finding a perpendicular line, a normal, or a perpendicular bisector

Scan to practise

📖 Quadratics: $x = -\frac{b}{2a}$, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $\Delta = b^2 - 4ac$

★ **$\Delta > 0$: two real roots; $\Delta = 0$: repeated root; $\Delta < 0$: complex conjugate roots** → Count real roots and find unknown constants

📖 Sum, product of roots of $\sum_0^n a_r x^r$: $-\frac{a_{n-1}}{a_n}$; $\frac{(-1)^n a_0}{a_n}$

★ **Factor & remainder theorems:** $p(a) = 0 \iff (x-a)$ **is a factor; remainder of** $p(x) \div (x-a)$ **is** $p(a)$ → Finding unknown coefficients and factorising cubics

★ **Odd; even:** $f(-x) = -f(x)$; $f(-x) = f(x)$ → Identify even and odd functions

★ **Composite; inverse:** $(f\circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$ → Finding $f^{-1}(x)$: swap $x$ and $y$, then make $y$ the subject
Watch for: $f^{-1}(a)$ means solve $f(x) = a$

★ **Transformations: $f(x)+b$ up $b$; $f(x-a)$ right $a$; $pf(x)$ vertical $\times p$; $f(qx)$ horizontal $\times\frac{1}{q}$; $-f(x)$, $f(-x)$ reflect in $x$-, $y$-axis** → Sketch translations and stretches, tracking key points
Watch for: Write $a + \frac{b}{x-c}$ first: each of $a$, $b$, $c$ is one transformation

★ **Graphs: $|f(x)|$ reflects negative parts up; $f(|x|)$ mirrors $x \geq 0$; $\frac{1}{f(x)}$ has asymptotes at zeros of $f$** → Sketch $|f(x)|$, $f(|x|)$, $\frac{1}{f(x)}$ and $[f(x)]^2$, tracking roots and asymptotes
Watch for: For $[f(x)]^2$, zeros become touching points

★ **$y = \frac{ax+b}{cx+d}$ asymptotes: $x = -\frac{d}{c}$, $y = \frac{a}{c}$** → Sketching reciprocal-type graphs; domains and ranges
Watch for: Asymptotes are equations: $y = \ldots$, not "$HA = \ldots$"

**Watch for** Write $a + \frac{b}{x-c}$ first: each part is one transformation

**Calculator** find intersections and max/min; solve equations: keystrokes at ibmathrevision.com/calculator-skills

## 3 Geometry & trigonometry

📖 3D distance ($\Delta x = x_1 - x_2$, ...): $d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

📖 3D midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

📖 Pyramid; cone; sphere: $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, $A = \pi rl$; $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$

Scan to practise

📖 Sine, cosine rules; area: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, $c^2 = a^2 + b^2 - 2ab\cos C$, $A = \frac{1}{2}ab\sin C$

📖 Arc; sector (radians): $l = r\theta$, $A = \frac{1}{2}r^2\theta$

📖 Identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos^2\theta + \sin^2\theta = 1$

📖 Reciprocal ratios: $\sec\theta = \frac{1}{\cos\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

📖 Pythagorean: $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

📖 Compound angles: $\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$, $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

📖 Compound tan: $\tan(A\pm B) = \frac{\tan A \pm \tan B}{1\mp\tan A\tan B}$

📖 Double angle: $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$, $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

★ **Exact values** $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$: $\sin: 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$; $\cos$: reverse; $\tan: 0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$, undef. → Every non-calculator trig question

★ **Ranges:** $\arcsin x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\arccos x \in [0, \pi]$, $\arctan x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ → Inverse trig functions with their domains and ranges

★ **$a\sin(b(x-c))+d$, $b>0$: amplitude $|a|$, period $\frac{2\pi}{b}$** → Matching a graph to its equation, and modelling

📖 Vector magnitude: $|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

📖 Scalar product: $\boldsymbol{v}\cdot\boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3 = |\boldsymbol{v}||\boldsymbol{w}|\cos\theta$

★ **Perpendicular; unit vector;** $\overrightarrow{AB}$: $\boldsymbol{v}\cdot\boldsymbol{w} = 0$; $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$; $\boldsymbol{b}-\boldsymbol{a}$ → Test whether two vectors are perpendicular; find unit vectors

📖 Line: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$; $x = x_0 + \lambda l$, ...; $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$

📖 Vector product: $\boldsymbol{v}\times\boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$, $|\boldsymbol{v}\times\boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}|\sin\theta$

📖 Area of parallelogram: $A = |\boldsymbol{v}\times\boldsymbol{w}|$

★ **Area of triangle:** $\frac{1}{2}|\boldsymbol{v}\times\boldsymbol{w}|$ → 3D triangle areas (the booklet gives the parallelogram)

📖 Plane: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$, $\boldsymbol{r}\cdot\boldsymbol{n} = \boldsymbol{a}\cdot\boldsymbol{n}$, $ax + by + cz = d$

★ **Angle line–plane; plane–plane:** $\sin\theta = \frac{|\boldsymbol{b}\cdot\boldsymbol{n}|}{|\boldsymbol{b}||\boldsymbol{n}|}$; $\cos\theta = \frac{|\boldsymbol{n}_1\cdot\boldsymbol{n}_2|}{|\boldsymbol{n}_1||\boldsymbol{n}_2|}$ → Angles between lines and planes

★ **Normal to plane through $\boldsymbol{a}$ with directions $\boldsymbol{b}, \boldsymbol{c}$:** $\boldsymbol{n} = \boldsymbol{b}\times\boldsymbol{c}$ → Normal vector found with the vector product

★ **Line of intersection of two planes: direction** $\boldsymbol{n}_1\times\boldsymbol{n}_2$ → Line of intersection of two planes

**Watch for** Factorise, never divide by a trig function: dividing loses solutions

**Calculator** do vectors; solve equations: keystrokes at ibmathrevision.com/calculator-skills

## 4 Statistics & probability

📖 IQR; mean: $IQR = Q_3 - Q_1$, $\bar{x} = \frac{\sum f_i x_i}{n}$

★ **Outliers:** $< Q_1 - 1.5\,IQR$ **or** $> Q_3 + 1.5\,IQR$ → Box plots and "is this value an outlier?" questions

★ **Variance:** $\sigma^2 = \frac{\sum f(x-\mu)^2}{n} = \frac{\sum fx^2}{n} - \mu^2$ → Variance and standard deviation as measures of spread

Scan to practise

📖 Probability: $P(A) = \frac{n(A)}{n(U)}$, $P(A) + P(A') = 1$

📖 Combined events: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

📖 Independent: $P(A\cap B) = P(A)P(B)$

📖 Conditional: $P(A \mid B) = \frac{P(A\cap B)}{P(B)}$

📖 Bayes: $P(B \mid A) = \frac{P(B)P(A|B)}{P(B)P(A|B)+P(B')P(A|B')}$

📖 Bayes, partition $B_1, B_2, B_3$: $P(B_i \mid A) = \frac{P(B_i)P(A|B_i)}{\sum_j P(B_j)P(A|B_j)}$

📖 Discrete $X$: $E(X) = \sum xP(X=x)$, $\operatorname{Var}(X) = E(X^2) - [E(X)]^2$

★ **Binomial pmf:** $P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$ → Fixed number of independent trials

📖 Binomial mean, variance: $np$, $np(1-p)$

📖 Standardised normal: $z = \frac{x-\mu}{\sigma}$

📖 Continuous $X$: $E(X) = \int x\,f(x)\,dx$, $\operatorname{Var}(X) = \int x^2 f(x)\,dx - \mu^2$

★ **pdf; median $m$:** $\int f(x)\,dx = 1$; $\int_{-\infty}^{m} f(x)\,dx = \frac{1}{2}$; **mode: max of $f$** → Continuous random variable questions
Watch for: Mode: where $f(x)$ is greatest on the domain, including endpoints

📖 Linear transformation: $E(aX+b) = aE(X) + b$, $\operatorname{Var}(aX+b) = a^2\operatorname{Var}(X)$

★ **Regression: $y$ on $x$ to predict $y$, $x$ on $y$ for $x$; interpolate only** → Choose the right regression line for a prediction

**Watch for** $E(X^2) = \Sigma x^2 P(X=x)$: square the values

**Calculator** do a normal distribution; do binomial and Poisson; find mean, SD and quartiles; do regression: keystrokes at ibmathrevision.com/calculator-skills

## 5 Calculus

📖 First principles: $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

📖 Basic derivatives: $(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$

📖 Trig derivatives: $(\tan x)' = \sec^2 x$, $(\sec x)' = \sec x\tan x$, $(\operatorname{cosec} x)' = -\operatorname{cosec} x\cot x$, $(\cot x)' = -\operatorname{cosec}^2 x$

Scan to practise

📖 More derivatives: $(a^x)' = a^x\ln a$, $(\log_a x)' = \frac{1}{x\ln a}$

📖 Inverse trig: $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$, $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$, $(\arctan x)' = \frac{1}{1+x^2}$

📖 Chain, product, quotient: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, $(uv)' = uv' + vu'$, $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$

★ **Implicit:** $\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$, $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ → Gradients of curves you cannot write as $y = \ldots$

★ **Related rates:** $\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt}$ → Link rates of change, such as a growing balloon

★ **Tangent; normal:** $y - f(a) = f'(a)(x-a)$; $m_n = -\frac{1}{f'(a)}$ → Any "find the equation of the tangent / normal" question

★ **Max / min; inflexion:** $f' = 0$ **and** $f'' < 0$ / $f'' > 0$; $f'' = 0$ **with sign change** → Optimisation and curve sketching

★ **L'Hôpital, for $\frac{0}{0}$ or $\frac{\infty}{\infty}$:** $\lim\frac{f(x)}{g(x)} = \lim\frac{f'(x)}{g'(x)}$ → Limits that give $\frac{0}{0}$

📖 Standard integrals: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $\int\frac{1}{x}\,dx = \ln|x| + C$, $\int\sin x\,dx = -\cos x + C$, $\int\cos x\,dx = \sin x + C$, $\int e^x\,dx = e^x + C$

📖 More integrals: $\int a^x\,dx = \frac{a^x}{\ln a} + C$, $\int\frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$, $\int\frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

★ **Learn:** $\int\sec^2 x\,dx = \tan x + C$, $\int\frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$, $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$ → Any fraction whose top is (a multiple of) the derivative of the bottom

★ **$\sin^2 x$, $\cos^2 x$:** $\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ → Integrating even powers of sin and cos

📖 By parts: $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

★ **Substitution:** $\int f(g(x))g'(x)\,dx = \int f(u)\,du$; **change the limits too** → Integrate by substitution, changing the limits

📖 Areas: $\int_a^b |y|\,dx$, $\int_c^d |x|\,dy$

📖 Volumes of revolution: $V = \int_a^b \pi y^2\,dx$, $V = \int_c^d \pi x^2\,dy$

📖 Kinematics: $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$; **distance** $\int|v|\,dt$

📖 Euler's method: $y_{n+1} = y_n + h\,f(x_n, y_n)$, $x_{n+1} = x_n + h$

★ **Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int\frac{dy}{g(y)} = \int f(x)\,dx$ → Solve by separating the variables, with an initial condition

★ **Homogeneous** $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$: $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$ → First-order equations where every term has the same degree

📖 Integrating factor for $y' + P(x)y = Q(x)$: $e^{\int P(x)\,dx}$

★ **Then:** $ye^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx$ → Solve linear differential equations with an integrating factor

📖 Maclaurin: $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$

📖 Series: $e^x = 1 + x + \frac{x^2}{2!} + \cdots$, $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

📖 sin, cos: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$, $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

📖 arctan: $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$

**Watch for** Radians for calculus: check the GDC mode

**Calculator** find derivatives and integrals; find intersections and max/min: keystrokes at ibmathrevision.com/calculator-skills



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