Voronoi Diagrams: exam practice

10 questions · 48 marks · worked solutions and markschemes

Name _____ Date _____ Score / 48

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/voronoi-diagrams-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

Three shops are at $A(1, 1)$, $B(7, 2)$ and $C(4, 7)$. Each customer goes to the nearest shop.

A customer lives at $(3, 5)$. Determine which shop they go to. **[2]**

2. **4 marks · Paper 1 style · calculator allowed**

Find the equation of the perpendicular bisector of the sites $P(2, 3)$ and $Q(8, 7)$, in the form $y = mx + c$. **[4]**

3. **4 marks · Paper 1 style · calculator allowed**

The perpendicular bisector of the sites $A(1, 2)$ and B has equation $y = -x + 7$.

Find the coordinates of B. **[4]**

4. **4 marks · Paper 1 style · calculator allowed**

Two phone masts are at $A(2, 2)$ and $B(8, 2)$ inside a square region $0 \leq x \leq 10$, $0 \leq y \leq 10$.

(a) Write down the equation of the edge between the two Voronoi cells. **[1]**

(b) Find the area of the cell of mast A inside the region. **[1]**

(c) Find the distance from the point $(5, 9)$ to the nearest mast. **[2]**

5. **4 marks · Paper 2 style · calculator allowed**

Three hospitals are at $A(0, 0)$, $B(8, 0)$ and $C(0, 6)$. A new clinic is to be placed inside triangle ABC as far as possible from the three hospitals.

(a) Find the position of the clinic. **[3]**

(b) Find its distance from each hospital. **[1]**

6. **5 marks · Paper 1 style · calculator allowed**

Three sites are at $A(0, 0)$, $B(6, 0)$ and $C(0, 4)$.

(a) Write down the equations of the perpendicular bisectors of $[AB]$ and $[AC]$. **[2]**

(b) Hence write down the coordinates of the vertex of the Voronoi diagram. **[1]**

(c) Determine which site is closest to the point $(5, 1)$. **[2]**

7. **7 marks · Paper 1 style · calculator allowed**

The perpendicular bisector of $[PQ]$ has equation $3x + 2y = 26$. The point P is $(1, 5)$.

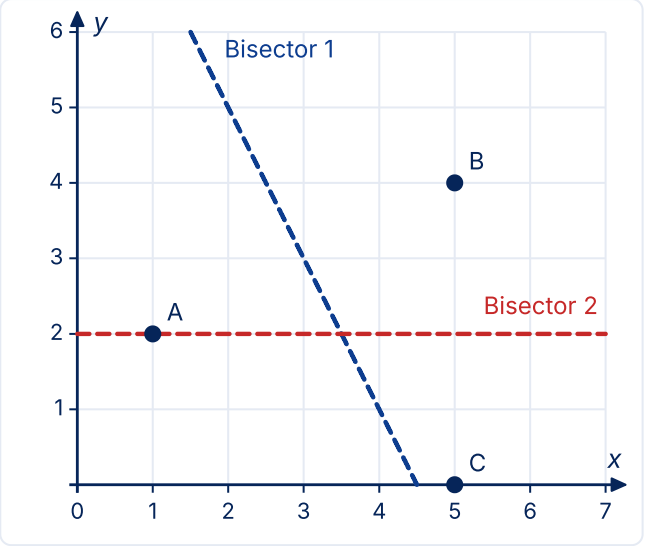
a. Find the gradient of $[PQ]$. **[2]**

b. Find the coordinates of the midpoint M of $[PQ]$. **[3]**

c. Find the coordinates of Q. **[2]**

8. **5 marks · Paper 2 style · calculator allowed**

Two sites, $A(1, 2)$ and $B(5, 4)$, have perpendicular bisector $y = -2x + 9$. A third site, $C(5, 0)$, is added to form a Voronoi diagram. The perpendicular bisector of [BC] is the horizontal line $y = 2$.



1. Find the coordinates of the Voronoi vertex, V, where the two bisectors meet.
2. A toxic waste dump is to be built at point V so it is as far away as possible from towns A, B, and C. Calculate the exact distance from the toxic waste dump to town A.

9. **6 marks · Paper 2 style · calculator allowed**

Three towns are at $A(0, 0)$, $B(10, 0)$ and $C(4, 8)$, with distances in km. A waste site is to be built inside triangle ABC as far as possible from all three towns.

(a) Find the coordinates of the point that is the same distance from all three towns. **[4]**

(b) Find the distance from this point to each town. **[2]**

10. **7 marks · Paper 2 style · calculator allowed**

Three villages are located at $A(0, 0)$, $B(8, 2)$ and $C(2, 10)$ (units in km). A mobile phone mast is to be placed equidistant from all three villages.

a. Find the equation of the perpendicular bisector of $[AB]$. **[2]**

b. Find the equation of the perpendicular bisector of $[AC]$. **[2]**

c. Find the coordinates of the mast. **[2]**

d. Find the distance from the mast to each village. **[1]**

Answers and worked solutions

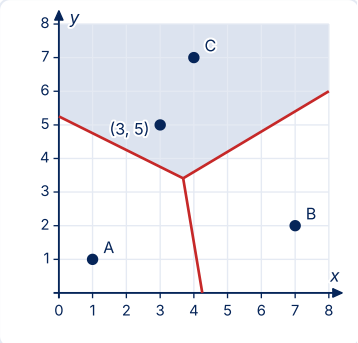
Question 1

2 marks

Answer: shop C

$AP = \sqrt{20}$, $BP = 5$, $CP = \sqrt{5}$ (M1)

shop C A1



(3, 5) is in the cell of C

Question 2

4 marks

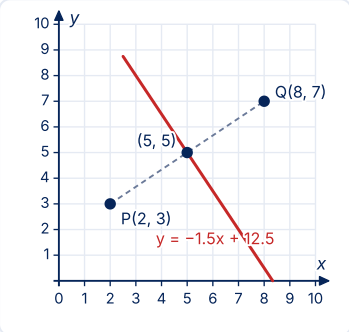
Answer: $y = -1.5x + 12.5$

midpoint $(5, 5)$ A1

gradient of $PQ = \frac{2}{3}$ A1

perpendicular gradient $-\frac{3}{2}$ (M1)

$y = -1.5x + 12.5$ A1



Through the midpoint (5, 5), gradient -1.5

Question 3

4 marks

Answer: $(5, 6)$

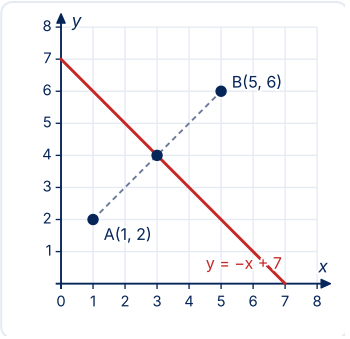
$[AB]$ has gradient 1:

$B = (1 + t,\ 2 + t)$ (M1)

midpoint $\left(1 + \frac{t}{2},\ 2 + \frac{t}{2}\right)$ lies on $y = -x + 7$ M1

$t = 4$ A1

$B(5, 6)$ A1



B is the mirror image of A

Question 4

4 marks

Answer: (a) $x = 5$; (b) 50; (c) 7.62

(a)

$x = 5$ A1

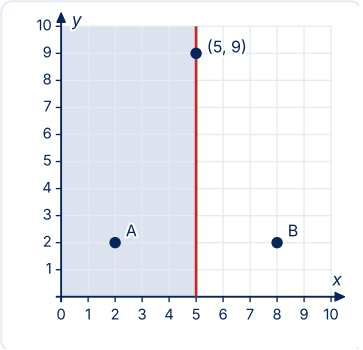
(b)

$5 \times 10 = 50$ A1

(c)

$\sqrt{3^2 + 7^2}$ (M1)

7.62 A1



Edge $x = 5$; cell of A has area 50

Question 5

4 marks

Answer: (a) $(4, 3)$; (b) 5

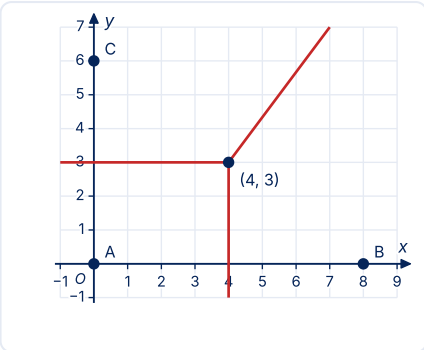
(a)

bisectors $x = 4$ and $y = 3$ (M1) (A1)

$(4, 3)$ A1

(b)

5 A1



Vertex (4, 3), 5 from each hospital

Question 6

5 marks

Answer: (a) $x = 3$ and $y = 2$; (b) $(3, 2)$; (c) B

(a)

$x = 3$ A1

$y = 2$ A1

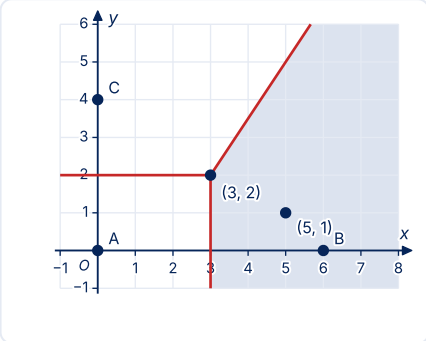
(b)

$(3, 2)$ A1

(c)

distances $\sqrt{26}$, $\sqrt{2}$, $\sqrt{34}$ (M1)

B A1



Vertex (3, 2); (5, 1) is nearest B

Question 7

7 marks

Answer: (a) $2/3$; (b) $(4, 7)$; (c) $(7, 9)$

a. bisector gradient $-\frac{3}{2}$ A1

PQ gradient $\frac{2}{3}$ A1

b. line through P:

$y - 5 = \frac{2}{3}(x - 1)$ **M1**

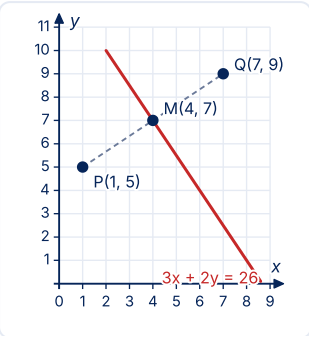
intersect with

$3x + 2y = 26$ **M1**

$M(4, 7)$ **A1**

c. $Q = 2M - P$ **(M1)**

$Q(7, 9)$ **A1**



Q is the mirror image of P

Question 8

5 marks

Answer: (a) $(3.5, 2)$; (b) 2.5

1. Set the two bisector equations equal:

   $-2x + 9 = 2$ **M1**

   Solve:

   $2x = 7 \implies x = 3.5$ **A1**

   Coordinates of the vertex:

   $V = (3.5, 2)$ **A1**

2. Use the distance formula from $V(3.5, 2)$ to $A(1, 2)$:

   $VA = \sqrt{(3.5 - 1)^2 + (2 - 2)^2}$ **M1**

   $VA = \sqrt{2.5^2} = 2.5$ units **A1**

Question 9

6 marks

Answer: (a) $(5,\ 2.5)$; (b) 5.59 km

(a)

perpendicular bisector of $[AB]$:

$x = 5$ **A1**

perpendicular bisector of $[AC]$:

$x + 2y = 10$ **A1**

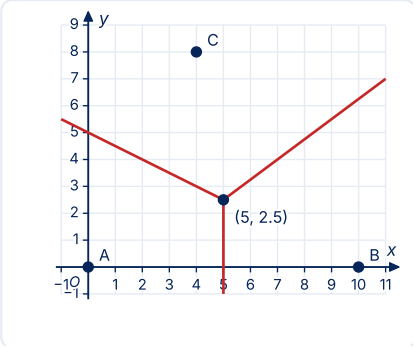
solving **(M1)**

$(5,\ 2.5)$ **A1**

(b)

$\sqrt{5^2 + 2.5^2}$ **(M1)**

5.59 km **A1**



Vertex (5, 2.5): 5.59 km from each town

Question 10

7 marks

Answer: (a) $y = -4x + 17$; (b) $y = -x/5 + 26/5$; (c) $(59/19, 87/19)$; (d) 5.53

a. midpoint $(4, 1)$, gradient of

$AB = \frac{1}{4}$ **A1**

$y = -4x + 17$ **A1**

b. midpoint $(1, 5)$, gradient of

$AC = 5$ **A1**

$y = -\frac{1}{5}x + \frac{26}{5}$ **A1**

c. solve simultaneously **(M1)**

$\left(\frac{59}{19}, \frac{87}{19}\right)$

$\approx (3.11, 4.58)$ **A1**

d. 5.53 km **A1**

Every solution is also on ibmathrevision.com/slai/voronoi-diagrams-exam-practice
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