Statistical Testing: exam practice

10 questions · 43 marks · worked solutions and markschemes

Name _____ Date _____ Score / 43

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/statistical-testing-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

A 3×3 contingency table has a grand total of 150 entries. A χ^2 test for independence is conducted. The expected frequency for the cell in the 2nd row and 3rd column is calculated to be 12. Given that the sum of the observed frequencies in the entire 2nd row is 45, find the sum of the observed frequencies in the entire 3rd column.

2. **3 marks · Paper 1 style · calculator allowed**

A contingency table has 3 rows and 4 columns. The total of row 2 is 60, the total of column 3 is 45 and the grand total is 150.

(a) Find the expected frequency for the cell in row 2, column 3. **[2]**

(b) Find the number of degrees of freedom. **[1]**

3. **3 marks · Paper 1 style · calculator allowed**

A goodness of fit test is carried out on data in 7 categories, using expected frequencies from a given distribution with no parameters estimated.

(a) Write down the number of degrees of freedom. **[1]**

(b) The p-value is 0.031. State the conclusion at the 5% level and at the 1% level. **[2]**

4. **4 marks · Paper 1 style · calculator allowed**

A χ^2 test for independence on a 3×2 table gives $\chi^2 = 7.21$. The test is at the 5% level and the critical value is 5.991.

(a) Write down the number of degrees of freedom. **[1]**

(b) State the conclusion, giving a reason. **[2]**

(c) Find the p-value of the test statistic. **[1]**

5. **4 marks · Paper 2 style · calculator allowed**

The number of accidents at a factory on each weekday over a year is shown. Test at the 5% level whether accidents are equally likely on each day.

Day	Mon	Tue	Wed	Thu	Fri
Accidents	28	20	18	22	32

(a) State the hypotheses. **[1]**

(b) Find the p-value. **[2]**

(c) State the conclusion. **[1]**

6. **5 marks · Paper 1 style · calculator allowed**

A university canteen sells baskets of candies. Each basket contains a collectible sticker featuring a famous mathematician. The canteen claims the stickers are distributed according to the following percentages:

Mathematician	Euler	Pythagoras	Lagrange	Laplace	Fourier
Percentages (%)	50	20	15	10	5

A staff member buys 140 baskets of candy for his students.

1. Calculate the expected frequency of finding a Pythagoras sticker.
2. Calculate the expected frequency of finding a Fourier sticker.
3. State the number of degrees of freedom for a χ^2 goodness-of-fit test.

7. **5 marks · Paper 2 style · calculator allowed**

A die is rolled 120 times to test whether it is fair. A χ^2 goodness of fit test is carried out at the 5% level.

Score	1	2	3	4	5	6
Frequency	15	22	18	25	17	23

(a) Write down the expected frequency for each score. **[1]**

(b) Write down the number of degrees of freedom. **[1]**

(c) Find the p-value and state the conclusion. **[3]**

8. **5 marks · Paper 2 style · calculator allowed**

A company claims that 50% of its customers choose small, 30% medium and 20% large cups. In a sample of 200 customers, 110 chose small, 50 medium and 40 large.

(a) Find the expected frequencies. **[2]**

(b) Find the χ^2 statistic and the p-value. **[2]**

(c) State, with a reason, whether the claim is supported at the 5% level. **[1]**

9. **6 marks · Paper 2 style · calculator allowed**

A survey asks 120 people whether they shop mostly online or in shops. The results are shown. A χ^2 test for independence is carried out at the 5% significance level.

	Online	In shops
Under 30	38	22
30 and over	27	33

(a) Write down the null hypothesis. **[1]**

(b) Find the expected frequency for under-30s who shop online. **[2]**

(c) Find the χ^2 statistic and the p-value. **[2]**

(d) State the conclusion of the test. **[1]**

10. **5 marks · Paper 2 style · calculator allowed**

200 people were asked which of three films they liked best. The results by gender are shown. A χ^2 test for independence is carried out at the 1% significance level.

	Film A	Film B	Film C
Group 1	25	30	45
Group 2	35	40	25

(a) Find the expected frequency for Group 1 and Film C. **[2]**

(b) Find the p-value. **[2]**

(c) State the conclusion of the test. **[1]**

Answers and worked solutions

Question 1

3 marks

Answer: 40

Expected frequency:
$E = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$
Substitute: $12 = \frac{45 \times C_3}{150}$ M1
Rearrange: $C_3 = \frac{12 \times 150}{45}$ A1
$C_3 = 40$ A1

Question 2

3 marks

Answer: (a) 18; (b) 6

(a)
$\frac{60 \times 45}{150}$ (M1)
18 A1

(b)
$(3-1)(4-1) = 6$ A1

Question 3

3 marks

Answer: (a) 6

(a)
6 A1

(b)
5%: reject H_0 A1
1%: do not reject H_0 A1

Question 4

4 marks

Answer: (a) 2; (c) 0.0272

(a)
$(3-1)(2-1) = 2$ A1

(b)
$7.21 > 5.991$ R1
reject H_0: the variables are not independent A1

(c)
0.0272 A1

Question 5

4 marks

Answer: (b) $p = 0.225$

(a)
H_0: accidents are equally likely on each day; H_1: they are not A1

(b)
expected 24 each (M1)
$p = 0.225$ A1

(c)
$0.225 > 0.05$: do not reject H_0 R1

Question 6

5 marks

Answer: (a) 28; (b) 7; (c) 4

1. Pythagoras is 20%. Expected $= 0.20 \times 140$. (M1)
   $= 28$ stickers. A1
2. Fourier is 5%. Expected $= 0.05 \times 140$. (M1)
   $= 7$ stickers. A1
3. $\text{df} = 5 - 1 = 4$. A1

Question 7

5 marks

Answer: (a) 20; (b) 5; (c) $p = 0.579$; no evidence of bias

(a)
20 A1

(b)
5 A1

(c)
$\chi^2 = 3.8$ (A1)
$p = 0.579$ A1
$0.579 > 0.05$: do not reject H_0; no evidence the die is biased R1

Question 8

5 marks

Answer: (a) 100, 60, 40; (b) $\chi^2 = 2.67$, $p = 0.264$

(a)
100, 60 A1
40 A1

(b)
$\chi^2 = 2.67$ A1
$p = 0.264$ A1

(c)
$0.264 > 0.05$, so do not reject H_0: the data are consistent with the claim R1

Question 9

6 marks

Answer: (b) 32.5; (c) $\chi^2 = 4.06$, $p = 0.0439$

(a)
H_0: where people shop is independent of age group A1

(b)
$\frac{60 \times 65}{120}$ (M1)
32.5 A1

(c)
$\chi^2 = 4.06$ A1
$p = 0.0439$ A1

(d)
$0.0439 < 0.05$, so reject H_0: there is evidence that where people shop depends on age group R1

Question 10

5 marks

Answer: (a) 35; (b) $p = 0.0122$

(a)
$\frac{100 \times 70}{200}$ (M1)
35 A1

(b)
$\chi^2 = 8.81$ (A1)
$p = 0.0122$ A1

(c)
$0.0122 > 0.01$: do not reject H_0 at the 1% level R1

Every solution is also on ibmathrevision.com/slai/statistical-testing-exam-practice
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