Sinusoidal Modelling: exam practice

10 questions · 42 marks · worked solutions and markschemes

Name _____ Date _____ Score / 42

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/sinusoidal-modelling-exam-practice

1. **4 marks · Paper 1 style · calculator allowed**

The depth of water in a harbour, D metres, t hours after midnight is modelled by $D(t) = a\cos(bt)° + d$. The greatest depth is 14 m at $t = 0$ and the next least depth is 4 m at $t = 6$.

Find the values of a, b and d. **[4]**

2. **3 marks · Paper 2 style · calculator allowed**

Let $h(t) = 2\sin(45t)° + 3$, for $0 \le t \le 8$.

Use your calculator to solve $h(t) = 4$. **[3]**

3. **3 marks · Paper 1 style · calculator allowed**

The height of a point on a rotating wheel, h in metres above the ground, is modelled by the function $h(t) = 5\sin(30t)° + 12$, where t is the time in seconds.

1. Write down the amplitude of the function.
2. Write down the equation of the principal axis.
3. State the maximum height reached by the point.

4. **4 marks · Paper 1 style · calculator allowed**

The height of a buoy, h metres, t seconds after it is observed is $h(t) = 3\sin(30t)° + 5$.

(a) Write down the amplitude. **[1]**

(b) Find the period. **[1]**

(c) Write down the maximum and minimum heights. **[2]**

5. **3 marks · Paper 2 style · calculator allowed**

The number of hours of daylight in a city on day t of the year is modelled by $L(t) = 12 + 4\sin\left(0.986(t - 80)\right)°$.

(a) Write down the greatest number of hours of daylight. **[1]**

(b) Find the number of hours of daylight on day 30. **[2]**

6. **4 marks · Paper 2 style · calculator allowed**

The number of daylight hours, L, in a city on day n of the year is modelled by $L(n) = 12 + 3.5\sin(0.986n - 80)°$.

1. Find the maximum number of daylight hours in a single day according to this model.
2. Using your GDC, estimate the first day of the year (value of n) when there are exactly 14 hours of daylight.

7. **4 marks · Paper 2 style · calculator allowed**

The temperature in a town, $T\,°\text{C}$, t hours after midnight is $T(t) = 8\cos(15(t - 14))° + 18$.

(a) Write down the maximum temperature and the time it occurs. **[2]**

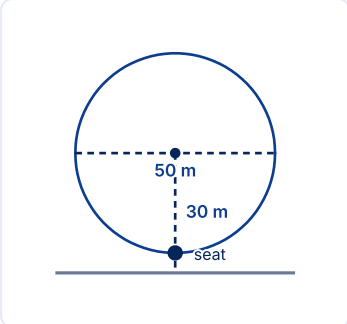
(b) Find the temperature at 09:00. **[2]**

8. **5 marks · Paper 2 style · calculator allowed**

A Ferris wheel has a diameter of 50 m and its centre is 30 m above the ground. It turns once every 10 minutes. A seat starts at the lowest point.

(a) Show that the height of the seat after t minutes can be written as $h(t) = 30 - 25\cos(36t)°$. **[3]**

(b) Find the height of the seat after 3 minutes. **[2]**



Diameter 50 m; centre 30 m up

9. **6 marks · Paper 2 style · calculator allowed**

The tip of a windmill blade rotates in a circle. Its height above the ground, $h(t)$ in metres, is modelled by $h(t) = a\sin(bt)° + d$, where t is the time in seconds. The maximum height of the tip is $15\ \text{m}$ and the minimum height is $3\ \text{m}$. One complete rotation of the windmill blade takes exactly $12\ \text{seconds}$. At $t = 0$, the tip is at a height of $9\ \text{m}$ and is moving upwards.

1. Find the exact values of a, b, and d.
2. Calculate the height of the tip after exactly $4\ \text{seconds}$.

10. **6 marks · Paper 2 style · calculator allowed**

The depth of water, D in metres, in a harbour is modelled by the function $D(t) = a\cos(bt)° + d$, where t is the time in hours after high tide. High tide occurs at $t = 0$ with a maximum depth of $14\ \text{m}$. The next low tide occurs at $t = 6$ hours with a minimum depth of $6\ \text{m}$.

1. Find the exact values of a and d.
2. Find the value of b.
3. Using your Graphic Display Calculator, find the amount of time during the first 12 hours when the depth of the water is strictly greater than $12\ \text{m}$.

Answers and worked solutions

Question 1

4 marks

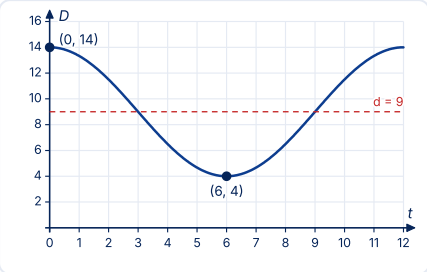
Answer: $a = 5,\ b = 30,\ d = 9$

$a = 5$ **A1**

$d = 9$ **A1**

period 12 hours **(M1)**

$b = 30$ **A1**



$a = 5,\ b = 30,\ d = 9$

Question 2

3 marks

Answer: $t = 0.667$ and $t = 3.33$

$\sin(45t)° = 0.5$ **(M1)**

$t = 0.667$ **A1**

$t = 3.33$ **A1**



Solutions $t = 0.667$ and 3.33

Question 3

3 marks

Answer: (a) 5; (b) $h = 12$; (c) 17

1. Amplitude $= 5$ **A1**
2. Principal axis is $h = 12$ (or $y = 12$) **A1**
3. Maximum height $= 12 + 5 = 17$ m **A1**

Question 4

4 marks

Answer: (a) 3; (b) 12 seconds; (c) $8\ m$ and $2\ m$

(a)

3 **A1**

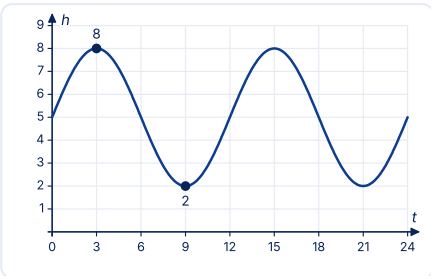
(b)

$\frac{360}{30} = 12$ seconds **A1**

(c)

$8\ m$ **A1**

$2\ m$ **A1**



Maximum 8 m, minimum 2 m, period 12 s

Question 5

3 marks

Answer: (a) 16 hours; (b) 8.97 hours

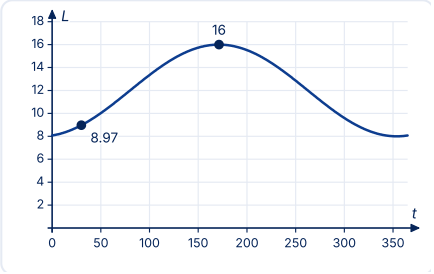
(a)

16 hours **A1**

(b)

$12 + 4\sin(0.986 \times (-50))°$ **(M1)**

8.97 hours **A1**



Maximum 16 hours; day 30: 8.97 hours

Question 6

4 marks

Answer: (a) 15.5; (b) 117

1. Maximum occurs when $\sin(0.986n - 80)° = 1$: $L = 12 + 3.5$ **(M1)** $= 15.5$ hours **A1**
2. Solve $12 + 3.5\sin(0.986n - 80)° = 14$ (graph $Y_2 = 14$ and find the first intersection) **(M1)** $n \approx 116.5$, so the first day is day 117 (accept $n \approx 116$ or 117) **A1**

Question 7

4 marks

Answer: (a) $26\ °\text{C}$ at 14:00; (b) $20.1\ °\text{C}$

(a)

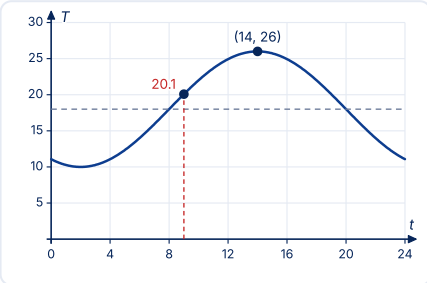
$26\ °\text{C}$ **A1**

at 14:00 **A1**

(b)

$8\cos(-75°) + 18$ **(M1)**

$20.1\ °\text{C}$ **A1**



Maximum (14, 26); $20.1°C$ at 09:00

Question 8

5 marks

Answer: (b) $37.7\ m$

(a)

amplitude 25, centre line 30 **A1**

$b = \frac{360}{10} = 36$ **A1**

starts at the minimum, so $-\cos$ **R1 AG**

(b)

$30 - 25\cos 108°$ **(M1)**

$37.7\ m$ **A1**

Question 9

6 marks

Answer: (a) a) 6; (a) b) 30; (a) d) 9; (b) 14.2

1. Principal axis:
   $d = \frac{15+3}{2} = 9$ **A1**
   Amplitude: $a = \frac{15-3}{2} = 6$ (positive, since at $t = 0$ the tip is on the axis moving upwards) **A1**
   Period $= 12$, so $b = \frac{360}{12}$ **(M1)**
   $b = 30$ **A1**
2. Substitute $t = 4$ into $h(t) = 6\sin(30t)° + 9$:
   $h(4) = 6\sin(120°) + 9$ **M1**
   $= 6\left(\frac{\sqrt{3}}{2}\right) + 9 = 9 + 3\sqrt{3} \approx 14.2\text{ m}$ **A1**

Question 10

6 marks

Answer: (a) a) 4; (a) d) 10; (b) 30; (c) 4

1. Amplitude:
   $a = \frac{\max - \min}{2} = \frac{14-6}{2} = 4$ **A1**
   Principal axis:
   $d = \frac{\max + \min}{2} = \frac{14+6}{2} = 10$ **A1**
2. High tide to low tide is half a period, so period $= 2 \times (6 - 0) = 12$ hours **(M1)**
   $b = \frac{360}{12} = 30$ **A1**
3. Require $4\cos(30t)° + 10 > 12$ for $0 \le t \le 12$.
   Graph $Y_1 = 4\cos(30x)° + 10$ and $Y_2 = 12$ and find the intersections **(M1)**
   Intersections at $t = 2$ and $t = 10$; the depth is above 12 m for $0 \le t < 2$ and $10 < t \le 12$.
   Total time $= 2 + 2 = 4$ hours **A1**

Every solution is also on ibmathrevision.com/slai/sinusoidal-modelling-exam-practice
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