Sequences and Series: exam practice

11 questions · 51 marks · worked solutions and markschemes

Name _____ Date _____ Score / 51

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/sequences-and-series-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

Find the value of $\sum_{k=1}^{30}(2k+5)$. **[2]**

2. **3 marks · Paper 1 style · calculator allowed**

Using the summation tool on your Graphic Display Calculator, evaluate the following arithmetic series:

$$\sum_{k=1}^{12}(5k-2)$$

Show the values you entered into your calculator.

3. **5 marks · Paper 1 style · calculator allowed**

The 1st, 3rd, and 7th terms of an arithmetic sequence (where the common difference $d \neq 0$) form the first three terms of a geometric sequence. Find the exact value of the common ratio, r, of the geometric sequence.

4. **4 marks · Paper 2 style · calculator allowed**

A gardener plants 12 trees in the first week, 15 in the second week, 18 in the third, and so on.

Find the number of weeks needed for the total number of trees planted to exceed 500. **[4]**

5. **4 marks · Paper 1 style · calculator allowed**

An arithmetic sequence has the following first three terms: $4, 9, 14, \ldots$

1. Find the common difference, d.
2. Find the value of the 15th term, u_{15}.
3. Find the sum of the first 15 terms of the sequence, S_{15}.

6. **4 marks · Paper 1 style · calculator allowed**

A job pays €$30\,000$ in the first year, and the pay rises by €1200 each year after that.

(a) Find the pay in the eighth year. **[2]**

(b) Find the total pay over the first 10 years. **[2]**

7. **5 marks · Paper 1 style · calculator allowed**

A geometric sequence is defined by the terms: $3, 12, 48, 192, \ldots$

1. Write down the common ratio, r.
2. Find the exact value of the 8th term, u_8.
3. Find the sum of the first 8 terms of the sequence, S_8.

8. **4 marks · Paper 2 style · calculator allowed**

The population of a village is 8000. It falls by 5% each year.

(a) Find the population after 6 years. **[2]**

(b) Find the number of complete years until the population is first below 5000. **[2]**

9. **5 marks · Paper 1 style · calculator allowed**

The fourth term of an arithmetic sequence is 18 and the ninth term is 43.

1. Find the common difference, d.
2. Find the first term, u_1.
3. Calculate the sum of the first 20 terms.

10. **7 marks · Paper 2 style · calculator allowed**

A geometric sequence has first term 5 and common ratio 1.2.

(a) Find the tenth term. **[2]**

(b) Find the sum of the first 10 terms. **[2]**

(c) Find the first term in the sequence that is greater than 100. **[3]**

11. **8 marks · Paper 2 style · calculator allowed**

In a geometric sequence, the second term is 12 and the fifth term is 40.5.

1. Find the common ratio, r, and the first term, u_1. **[3 marks]**
2. Find the sum of the first 10 terms. **[2 marks]**
3. Find the least number of terms that must be added for the sum to exceed $10\,000$. **[3 marks]**

Answers and worked solutions

Question 1

2 marks

Answer: 1080

$7 + 9 + \cdots + 65$, or GDC (M1)

1080 A1

Question 2

3 marks

Answer: 366

GDC summation entry: expression $5k - 2$, variable k, lower limit 1, upper limit 12. (M1)

(Check: arithmetic series with $n = 12$, $u_1 = 3$, $u_{12} = 58$, so $S_{12} = \frac{12}{2}(3 + 58)$.) (A1)

Sum $= 366$. A1

Question 3

5 marks

Answer: 2

Express the terms:

$u_1 = a,\ u_3 = a + 2d,\ u_7 = a + 6d$ (M1)

Equal common ratios: $\frac{a+2d}{a} = \frac{a+6d}{a+2d}$ M1

Cross-multiply and expand:

$a^2 + 4ad + 4d^2 = a^2 + 6ad$ A1

Simplify:

$4d^2 - 2ad = 0 \implies 2d(2d - a) = 0$; since $d \neq 0$, $a = 2d$ A1

Ratio: $r = \frac{2d+2d}{2d} = 2$ A1

Question 4

4 marks

Answer: 16 weeks

$S_n = \frac{n}{2}(24 + 3(n-1))$ (M1)

$\frac{n}{2}(3n + 21) > 500$ (A1)

$n > 15.08\ldots$ (A1)

16 weeks A1

Question 5

4 marks

Answer: (a) 5; (b) 74; (c) 585

1. $d = 9 - 4 = 5$. A1
2. Substitute into $u_n = u_1 + (n-1)d$:
   $u_{15} = 4 + 14(5)$. (M1)
   $u_{15} = 74$. A1
3. $S_{15} = \frac{15}{2}(4 + 74) = 7.5(78) = 585$. A1

Question 6

4 marks

Answer: (a) €$38\,400$; (b) €$354\,000$

(a)

$30\,000 + 7 \times 1200$ (M1)

€$38\,400$ A1

(b)

$\frac{10}{2}(2 \times 30\,000 + 9 \times 1200)$ (M1)

€$354\,000$ A1

Question 7

5 marks

Answer: (a) 4; (b) 49152; (c) 65535

1. $r = \frac{12}{3} = 4$. A1
2. $u_8 = 3(4)^{8-1} = 3(4)^7$. (M1)
   $u_8 = 49\,152$. A1
3. $S_8 = \frac{3(4^8-1)}{4-1} = \frac{3(65535)}{3}$. (M1)
   $S_8 = 65\,535$. A1

Question 8

4 marks

Answer: (a) 5880 (5881); (b) 10 years

(a)

8000×0.95^6 (M1)

5881 A1

(b)

$8000 \times 0.95^n < 5000$ (M1)

10 years A1

Question 9

5 marks

Answer: (a) 5; (b) 3; (c) 1010

1. $u_1 + 3d = 18$ and $u_1 + 8d = 43$ M1
   Subtracting the equations gives $5d = 25 \implies d = 5$ A1
2. $u_1 + 3(5) = 18 \implies u_1 = 3$ A1
3. $S_{20} = \frac{20}{2}(2(3) + 19(5))$ M1
   $S_{20} = 10(6 + 95) = 1010$ A1

Question 10

7 marks

Answer: (a) 25.8; (b) 130; (c) 111 (the 18th term)

(a)

5×1.2^9 (M1)

25.8 A1

(b)

$\frac{5(1.2^{10} - 1)}{0.2}$ (M1)

130 $(129.79\ldots)$ A1

(c)

$5 \times 1.2^{n-1} > 100$ (M1)

$n = 18$ A1

$u_{18} = 111$ A1

Question 11

8 marks

Answer: (a) r) 1.5; (a) u_1) 8; (b) 907; (c) 16

1. $u_1 r = 12$ and $u_1 r^4 = 40.5$, so
   $r^3 = \frac{40.5}{12} = 3.375$ M1
   $r = 1.5$ A1
   $u_1 = \frac{12}{1.5} = 8$ A1
2. $S_{10} = \frac{8\left(1.5^{10} - 1\right)}{1.5 - 1}$ M1
   $S_{10} = 907$ (906.640…) A1
3. $\frac{8\left(1.5^n - 1\right)}{0.5} > 10\,000$ M1
   Using the GDC: $S_{15} = 6990.3$, $S_{16} = 10\,493.5$ A1
   $n = 16$ A1

Every solution is also on ibmathrevision.com/slai/sequences-and-series-exam-practice
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