Quadratic Modelling: exam practice

10 questions · 42 marks · worked solutions and markschemes

Name _____ Date _____ Score / 42

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/quadratic-modelling-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

The quadratic model $y = a(x - 3)^2 + 8$ passes through the point $(1, 0)$. Find the value of a.

2. **3 marks · Paper 2 style · calculator allowed**

The height, h metres, of a ball t seconds after it is thrown is modelled by $h(t) = -5t^2 + 20t + 1.5$.

a. Find the height of the ball after 1 second. **[1]**

b. Find the maximum height of the ball. **[2]**

3. **4 marks · Paper 2 style · calculator allowed**

The height, h in metres, of a projectile fired upwards from a platform is given by the model $h(t) = -5t^2 + 20t + 10$, where t is the time in seconds since it was fired.

1. Find the time at which the projectile reaches its maximum height.
2. Calculate the maximum height reached by the projectile.

4. **4 marks · Paper 1 style · calculator allowed**

The shape of an arch is modelled by $y = -0.05x^2 + 2x$, where x and y are in metres and $y = 0$ is ground level.

(a) Find the width of the arch at ground level. **[2]**

(b) Find the maximum height of the arch. **[2]**

5. **4 marks · Paper 1 style · calculator allowed**

A rectangle has a perimeter of 60 cm and one side of length x cm.

(a) Show that the area is $A = 30x - x^2$. **[2]**

(b) Find the maximum area. **[2]**

6. **5 marks · Paper 2 style · calculator allowed**

The entrance of a railway tunnel is modelled by the quadratic function $y = -0.025x^2 + 1.5x$, where x is the horizontal distance from the left edge of the entrance, and y is the height above the ground. Both are measured in metres.

1. Find the horizontal width of the tunnel entrance at ground level.
2. Calculate the maximum height of the tunnel entrance.

7. **5 marks · Paper 2 style · calculator allowed**

The cross-section of a bridge arch is modelled by the quadratic function $y = -0.04x^2 + 1.6x$, where x is the horizontal distance from the left base of the arch, and y is the height above the ground. Both are measured in metres.

1. Find the horizontal width of the base of the arch.
2. Calculate the maximum height of the arch.

8. **4 marks · Paper 2 style · calculator allowed**

A quadratic model $y = ax^2 + bx + c$ fits the points $(0, 1.5)$, $(2, 5.1)$ and $(4, 6.3)$ exactly.

(a) Find the values of a, b and c. **[3]**

(b) Use the model to find the value of x where y is greatest. **[1]**

9. **5 marks · Paper 2 style · calculator allowed**

A water jet from a fountain follows a parabolic path modelled by the function $h(x) = -0.1x^2 + 1.2x + 1.5$, where h is the height of the water in metres, and x is the horizontal distance in metres from the nozzle.

1. Find the horizontal distance from the nozzle at which the water reaches its maximum height.
2. Calculate the maximum height reached by the water jet.

10. **6 marks · Paper 2 style · calculator allowed**

The daily profit P, in thousands of euros, from selling x thousand units of a product is modelled by $P(x) = -0.5x^2 + 12x - 15$.

1. Find the number of units that must be sold to maximize the daily profit.
2. Find the maximum daily profit.
3. Determine the break-even points where the profit is exactly zero. Give your answers to 3 significant figures.

Answers and worked solutions

Question 1

2 marks

Answer: $a = -2$

$a(1-3)^2 + 8 = 0$ (M1)

$a = -2$ A1

Question 2

3 marks

Answer: (a) $16.5\ m$; (b) $21.5\ m$

a. $16.5\ m$ A1

b. at $t = 2$ (M1)

$21.5\ m$ A1

Question 3

4 marks

Answer: (a) 2; (b) 30

1. Axis of symmetry:
   $t = \frac{-b}{2a} = \frac{-20}{2(-5)}$ M1
   $t = 2$ seconds A1
2. Substitute $t = 2$:
   $h(2) = -5(2)^2 + 20(2) + 10$ M1
   $h(2) = -20 + 40 + 10 = 30$ metres A1

Question 4

4 marks

Answer: (a) $40\ m$; (b) $20\ m$

(a)

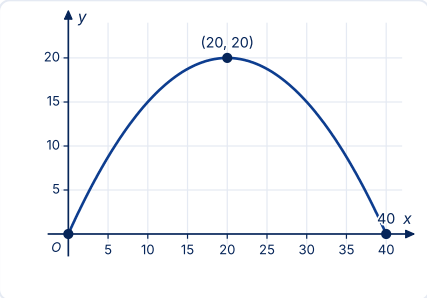
$x(2 - 0.05x) = 0$ (M1)

$40\ m$ A1

(b)

$x = 20$ (M1)

$20\ m$ A1



Width 40 m; height 20 m

Question 5

4 marks

Answer: (b) $225\ \text{cm}^2$

(a)

other side $30 - x$ M1

$A = x(30 - x)$ A1 AG

(b)

$x = 15$ (M1)

$225\ \text{cm}^2$ A1

Question 6

5 marks

Answer: (a) 60; (b) 22.5

1. Set $y = 0$ and factorise:
   $-0.025x^2 + 1.5x = 0 \implies x(-0.025x + 1.5) = 0$ M1
   $x = 0$ or $x = \frac{1.5}{0.025} = 60$
   Width is 60 m. A1
2. Axis of symmetry:
   $x = \frac{-1.5}{2(-0.025)} = 30$ M1
   Substitute $x = 30$:
   $y = -0.025(30)^2 + 1.5(30)$ M1
   $= -22.5 + 45 = 22.5$ m A1

Question 7

5 marks

Answer: (a) 40; (b) 16

1. Set $y = 0$ and factorise:
   $-0.04x^2 + 1.6x = 0 \implies x(-0.04x + 1.6) = 0$ M1
   $x = 0$ or $x = \frac{1.6}{0.04} = 40$
   Width is 40 m. A1
2. Axis of symmetry:
   $x = \frac{-1.6}{2(-0.04)} = 20$ M1
   Substitute $x = 20$:
   $y = -0.04(20)^2 + 1.6(20)$ M1
   $= -16 + 32 = 16$ m A1

Question 8

4 marks

Answer: (a) $a = -0.3,\ b = 2.4,\ c = 1.5$; (b) 4

(a)

$c = 1.5$, $4a + 2b = 3.6$,
$16a + 4b = 4.8$ (M1)(A1)

$a = -0.3,\ b = 2.4,\ c = 1.5$ A1

(b)

$x = 4$ A1

Question 9

5 marks

Answer: (a) 6; (b) 5.1

1. Axis of symmetry:
   $x = \frac{-b}{2a} = \frac{-1.2}{2(-0.1)} = \frac{-1.2}{-0.2}$ M1
   $x = 6$ m A1
2. Substitute $x = 6$:
   $h(6) = -0.1(6)^2 + 1.2(6) + 1.5$ M1
   $= -3.6 + 7.2 + 1.5$ M1
   $= 5.1$ m A1

Question 10

6 marks

Answer: (a) 12000; (b) 57000; (c) $1.32, 22.7$

1. Axis of symmetry:
   $x = \frac{-12}{2(-0.5)} = 12$ M1
   12 000 units must be sold. A1
2. Substitute $x = 12$:
   $P(12) = -0.5(144) + 144 - 15 = 57$
   Maximum profit is €$57\,000$. A1
3. Set $-0.5x^2 + 12x - 15 = 0$, i.e. $x^2 - 24x + 30 = 0$ M1
   Using GDC or the quadratic formula: $x = 12 \pm \sqrt{114}$
   $x \approx 1.32$ A1 and $x \approx 22.7$ A1
   (i.e. about 1320 units and 22 700 units)

Every solution is also on ibmathrevision.com/slai/quadratic-modelling-exam-practice
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