Probability Distributions: exam practice

10 questions · 49 marks · worked solutions and markschemes

Name _____ Date _____ Score / 49

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/probability-distributions-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

The probability that a flight arrives on time is 0.9. An airline records the arrivals of 75 flights. Calculate the probability that strictly more than 72 flights arrive on time.

2. **3 marks · Paper 2 style · calculator allowed**

A biased 6-sided die has a probability of 0.4 of landing on a six. The die is rolled 15 times.

Find the probability that the die lands on a six **at least** 6 times.

3. **3 marks · Paper 2 style · calculator allowed**

The marks in a test are normally distributed with standard deviation 5. 10% of students score more than 60.

Find the mean mark. **[3]**

4. **3 marks · Paper 1 style · calculator allowed**

A basketball player scores a free throw with probability 0.7, independently. She takes 12 free throws. Let M be the number she misses.

a. Write down the distribution of M. **[1]**

b. Find $P(M = 2)$. **[1]**

c. Find $P(M \leq 3)$. **[1]**

5. **5 marks · Paper 2 style · calculator allowed**

20% of seeds of a certain type fail to grow. 15 seeds are planted.

(a) Find the probability that exactly 3 seeds fail to grow. **[2]**

(b) Find the probability that at most 2 seeds fail to grow. **[2]**

(c) Write down the expected number of seeds that fail. **[1]**

6. **6 marks · Paper 2 style · calculator allowed**

The lengths of bolts are normally distributed with mean 42 mm and standard deviation 3 mm.

(a) Find the probability that a bolt is shorter than 40 mm. **[2]**

(b) Find the probability that a bolt is between 39 mm and 45 mm long. **[2]**

(c) Find the length that 95% of bolts are shorter than. **[2]**

7. **5 marks · Paper 2 style · calculator allowed**

The mass of a bag of rice is normally distributed with mean 500 g and standard deviation 6 g. A bag is labelled 'heavy' if its mass is more than 510 g.

(a) Find the probability that a bag is labelled heavy. **[2]**

(b) In a delivery of 50 bags, find the expected number labelled heavy. **[1]**

(c) Find the probability that at least one of the 50 bags is labelled heavy. **[2]**

8. **6 marks · Paper 2 style · calculator allowed**

The masses of harvested apples are normally distributed with mean 150 g and standard deviation 20 g.

1. Find the probability that a randomly selected apple has a mass between 140 g and 160 g. **[2 marks]**
2. The heaviest 10% of apples are sold as "premium". Find the minimum mass of a premium apple. **[2 marks]**
3. Two apples are chosen at random. Find the probability that both of them are premium. **[2 marks]**

9. **7 marks · Paper 2 style · calculator allowed**

The weights, W kg, of parcels delivered by a courier are normally distributed with mean 3.8 kg and standard deviation 1.2 kg.

1. Find the probability that a randomly chosen parcel weighs less than 1.5 kg. **[2 marks]**
2. The heaviest 8% of parcels are charged a surcharge. Find the minimum weight of a parcel that is charged the surcharge. **[2 marks]**
3. On one day the courier delivers 50 parcels. Find the expected number of these parcels that weigh less than 1.5 kg. **[1 mark]**
4. A random sample of 20 parcels is chosen. Find the probability that exactly 3 of them weigh less than 1.5 kg. **[2 marks]**

10. **8 marks · Paper 2 style · calculator allowed**

The masses of a particular type of apple are normally distributed with mean 125 g and standard deviation 18 g.

1. Find the probability that an apple has a mass less than 100 g. **[2 marks]**
2. Write down the probability that an apple has a mass greater than 150 g. **[1 mark]**
3. The middle 80% of apples have masses between a g and b g, where the interval is symmetric about the mean. Find a and b. **[3 marks]**
4. A bag contains 12 randomly chosen apples. Find the probability that at least 2 of them have a mass less than 100 g. **[2 marks]**

Answers and worked solutions

Question 1

3 marks

Answer: 0.0161

Let X be the number of flights on time: $X \sim B(75,\ 0.9)$ **M1**

"Strictly more than 72" means $P(X > 72) = P(X \geq 73) = 1 - P(X \leq 72)$ **(M1)**

Using the binomial CDF on the GDC:

$P(X \geq 73) = 0.016128\ldots \approx 0.0161$ **A1**

Question 2

3 marks

Answer: 0.597

Let X be the number of sixes, so $X \sim B(15,\ 0.4)$; we need $P(X \geq 6)$ **(M1)**

Use the complement:

$P(X \geq 6) = 1 - P(X \leq 5)$ **(M1)**

Binomial CDF on the GDC:

$1 - 0.40321\ldots = 0.597$ (3 s.f.) **A1**

Question 3

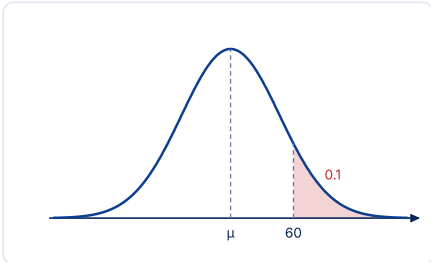
3 marks

Answer: $\mu = 53.6$

$P(X > 60) = 0.1$,

$z = 1.2815\ldots$ **(M1)(A1)**

$\mu = 53.6$ **A1**



$P(X > 60) = 0.1$

Question 4

3 marks

Answer: (a) $B(12, 0.3)$; (b) 0.168; (c) 0.493

a. $M \sim B(12, 0.3)$ **A1**

b. 0.168 **A1**

c. 0.493 **A1**

Question 5

5 marks

Answer: (a) 0.250; (b) 0.398; (c) 3

(a)

$X \sim B(15, 0.2)$ **(M1)**

0.250 **A1**

(b)

$P(X \leq 2)$ **(M1)**

0.398 **A1**

(c)

3 **A1**

Question 6

6 marks

Answer: (a) 0.252; (b) 0.683; (c) 46.9 mm

(a)

normal cdf **(M1)**

0.252 **A1**

(b)

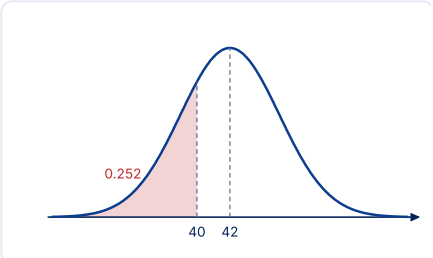
normal cdf **(M1)**

0.683 **A1**

(c)

inverse normal **(M1)**

46.9 mm **A1**



$P(\text{length} < 40) = 0.252$

Question 7

5 marks

Answer: (a) 0.0478; (b) 2.39; (c) 0.914

(a)

$P(M > 510)$ **(M1)**

0.0478 **A1**

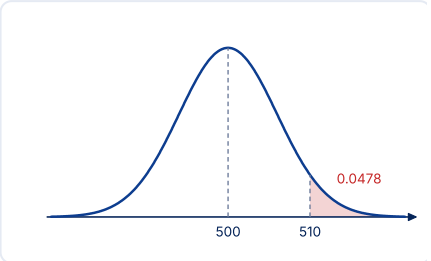
(b)

2.39 **A1**

(c)

$1 - (1 - 0.0477\ldots)^{50}$ **(M1)**

0.914 **A1**



$P(\text{mass} > 510) = 0.0478$

Question 8

6 marks

Answer: (a) 0.383; (b) 176; (c) 0.01

1. Normal cdf between 140 and 160 with $\mu = 150$, $\sigma = 20$ **(M1)**
   $= 0.383$ (0.38292…) **A1**
2. $P(M < k) = 0.9$; inverse normal **(M1)**
   $k = 176\ g(175.63\ldots)$ **A1**
3. The apples are independent: 0.1×0.1 **M1**
   $= 0.01$ **A1**

Question 9

7 marks

Answer: (a) 0.0276; (b) 5.49; (c) 1.38; (d) 0.0149

1. $P(W < 1.5)$ using the GDC normal cdf with $\mu = 3.8, \sigma = 1.2$ **(M1)**
   $= 0.0276$ (0.027640…)
   A1
2. $P(W > w) = 0.08 \Rightarrow P(W < w) = 0.92$; inverse normal **(M1)**
   $w = 5.49$ kg (5.4860…)
   A1
3. $50 \times 0.027640\ldots = 1.38$
   A1
4. $X \sim B(20,\ 0.027640\ldots)$, $P(X = 3)$ **(M1)**
   $= 0.0149$ (0.014948…)
   A1

Every solution is also on ibmathrevision.com/slai/probability-distributions-exam-practice
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Question 10

8 marks

Answer: (a) 0.0824; (b) 0.0824; (c) a) 102; (c) b) 148; (d) 0.260

1. Normal cdf, $\mu = 125$, $\sigma = 18$ **(M1)**
   $P(M < 100) = 0.0824$ (0.082433…) **A1**
2. By symmetry (150 is as far above the mean as 100 is below),
   $P(M > 150) = 0.0824$
   A1
3. $P(M < a) = 0.1$ and $P(M < b) = 0.9$ **M1**
   $a = 102$ $g(101.93\ldots)$ **A1**, $b = 148$
   $g(148.07\ldots)$ **A1**
4. $X \sim B(12,\ 0.082433\ldots)$:
   $P(X \geq 2) = 1 - P(X \leq 1)$ **(M1)**
   $= 0.260$ (0.25986…) **A1**