Optimisation: exam practice

10 questions · 40 marks · worked solutions and markschemes

Name _____ Date _____ Score / 40

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/optimisation-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

A cinema finds that at a ticket price of €p it sells $400 - 20p$ tickets.

Find the ticket price that gives the greatest income, and that income. **[3]**

2. **4 marks · Paper 1 style · calculator allowed**

The profit, €P, from making x items is $P(x) = -2x^2 + 120x - 1000$.

Find the number of items that gives the greatest profit, and the greatest profit. **[4]**

3. **3 marks · Paper 2 style · calculator allowed**

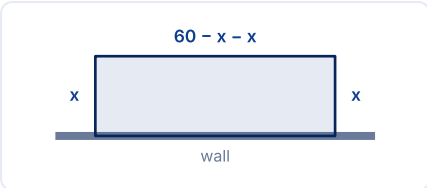
The concentration of a drug in the blood, C mg per litre, t hours after it is taken is $C(t) = \dfrac{6t}{t^2 + 4}$.

Use your calculator to find the greatest concentration and when it occurs. **[3]**

4. **4 marks · Paper 1 style · calculator allowed**

A farmer uses 60 m of fencing to make a rectangular pen against a long wall. The wall forms one side. The two sides at right angles to the wall are x m long.

Find the greatest area the pen can have. **[4]**



Fencing: x + x + (60 − x − x) = 60 m

5. **4 marks · Paper 2 style · calculator allowed**

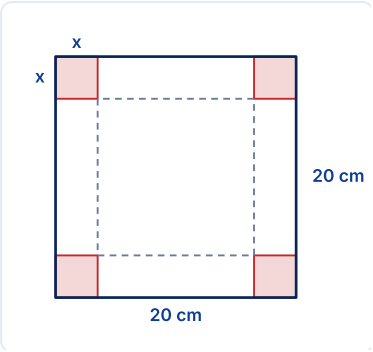
The cost of a lorry trip, €C, when it travels at a steady speed of v km h^{-1} is $C(v) = \dfrac{2000}{v} + 2v$.

Find the speed that gives the least cost, and the least cost. **[4]**

6. **4 marks · Paper 2 style · calculator allowed**

An open box is made from a square sheet of card 20 cm by 20 cm by cutting a square of side x cm from each corner and folding up the sides.

Find the value of x that gives the greatest volume, and the greatest volume. **[4]**



Cut squares of side x, fold along the dashes

7. **4 marks · Paper 1 style · calculator allowed**

A rectangle has a perimeter of 40 cm and a width of x cm.

(a) Show that its area is $A = 20x - x^2$. **[1]**

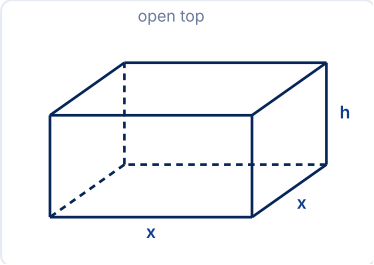
(b) Find the maximum area. **[3]**

8. **5 marks · Paper 2 style · calculator allowed**

An open box has a square base of side x cm and a volume of $32\ \text{cm}^3$.

(a) Show that the surface area is $A = x^2 + \dfrac{128}{x}$. **[2]**

(b) Find the minimum surface area. **[3]**



Square base x by x, height h, no lid

9. **4 marks · Paper 2 style · calculator allowed**

Two positive numbers have a sum of 20. One number is x.

(a) Write down an expression for P, the product of the square of x and the other number. **[1]**

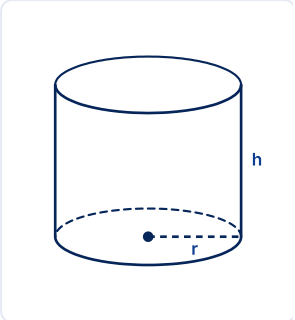
(b) Find the greatest value of P. **[3]**

10. **5 marks · Paper 2 style · calculator allowed**

A closed cylindrical can has volume $500\ \text{cm}^3$ and radius r cm.

(a) Show that its surface area is $A = 2\pi r^2 + \dfrac{1000}{r}$. **[2]**

(b) Use your calculator to find the radius that gives the least surface area, and the least surface area. **[3]**



Closed cylinder: radius r, height h

Answers and worked solutions

Question 1 — 3 marks

Answer: €10, €2000

$R = p(400 - 20p)$,
$R' = 400 - 40p = 0$ **M1**

$p = 10$ **A1**

€2000 **A1**

Question 2 — 4 marks

Answer: 30 items, €800

$P'(x) = -4x + 120$ **A1**

$-4x + 120 = 0$ **M1**

$x = 30$ **A1**

€800 **A1**

Question 3 — 3 marks

Answer: 1.5 mg per litre at $t = 2$ hours

GDC maximum **(M1)**

$t = 2$ hours **A1**

1.5 mg per litre **A1**

Question 4 — 4 marks

Answer: $450\ \text{m}^2$

$A = x(60 - 2x)$ **A1**

$60 - 4x = 0$ **M1**

$x = 15$ **A1**

$450\ \text{m}^2$ **A1**

Question 5 — 4 marks

Answer: 31.6 km h^{-1}, €126

$C'(v) = -\dfrac{2000}{v^2} + 2 = 0$ **M1**

$v^2 = 1000$ **(A1)**

$v = 31.6$ km h^{-1} **A1**

€126 **A1**

Question 6 — 4 marks

Answer: $x = 3.33$, $V = 593\ \text{cm}^3$

$V = x(20 - 2x)^2$ **A1**

GDC or $V'(x) = 0$ **(M1)**

$x = 3.33$ **A1**

$593\ \text{cm}^3$ **A1**

Question 7 — 4 marks

Answer: (b) $100\ \text{cm}^2$

(a)

length $20 - x$, so
$A = x(20 - x)$ **A1 AG**

(b)

$\dfrac{dA}{dx} = 20 - 2x = 0$ **M1**

$x = 10$ **A1**

$100\ \text{cm}^2$ **A1**

Question 8 — 5 marks

Answer: (b) $48\ \text{cm}^2$

(a)

height $\dfrac{32}{x^2}$ **M1**

$A = x^2 + 4x \times \dfrac{32}{x^2}$ **A1 AG**

(b)

$2x - \dfrac{128}{x^2} = 0$ **M1**

$x = 4$ **A1**

$48\ \text{cm}^2$ **A1**

Question 9 — 4 marks

Answer: (a) $P = x^2(20 - x)$; (b) 1190

(a)

$P = x^2(20 - x)$ **A1**

(b)

$P' = 40x - 3x^2 = 0$ **M1**

$x = \dfrac{40}{3}$ **A1**

1190 (1185.2) **A1**

Question 10 — 5 marks

Answer: (b) $r = 4.30$ cm, $349\ \text{cm}^2$

(a)

$h = \dfrac{500}{\pi r^2}$ **M1**

$A = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$ **A1 AG**

(b)

GDC minimum **(M1)**

$r = 4.30$ cm **A1**

$349\ \text{cm}^2$ **A1**

Every solution is also on ibmathrevision.com/slai/optimisation-exam-practice
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