Logarithms and Exponentials: exam practice

10 questions · 39 marks · worked solutions and markschemes

Name _____ Date _____ Score / 39

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/logarithms-and-exponentials-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

Solve $5^x = 80$. **[2]**

2. **2 marks · Paper 1 style · calculator allowed**

Solve $3\log_{10} x = 2$. **[2]**

3. **5 marks · Paper 1 style · calculator allowed**

The temperature of a cup of coffee, T in degrees Celsius, t minutes after being poured is given by the exponential model $T(t) = 20 + 75e^{-0.08t}$. Find the exact time required for the coffee to cool to $35°\text{C}$.

4. **4 marks · Paper 1 style · calculator allowed**

Using the logarithm functions on your Graphic Display Calculator, evaluate the following expressions:

1. $\log_{10} 1000$
2. $\ln(e^4)$
3. $e^{\ln 7}$

5. **4 marks · Paper 1 style · calculator allowed**

Use your GDC's equation solver or graphing tool to find the exact value of x for each of the following exponential equations. Give your answers correct to 3 significant figures.

1. $3^x = 20$
2. $e^{2x} = 18$

6. **4 marks · Paper 1 style · calculator allowed**

The pH of a solution is given by $\text{pH} = -\log_{10}[\text{H}^+]$, where $[\text{H}^+]$ is the concentration of hydrogen ions in moles per litre.

(a) Find the pH of a solution with $[\text{H}^+] = 3.2 \times 10^{-5}$. **[2]**

(b) A solution has a pH of 6.2. Find $[\text{H}^+]$. **[2]**

7. 4 marks · Paper 2 style · calculator allowed

The population of a town is modelled by $P(t) = 2400e^{0.035t}$, where t is the number of years after 2020.

(a) Find the population in 2030. **[2]**

(b) Find the number of years it takes for the population to double. **[2]**

8. 4 marks · Paper 2 style · calculator allowed

The loudness of a sound, L decibels, is given by $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where I is its intensity and I_0 is a fixed reference intensity.

(a) Find L when $I = 1000I_0$. **[2]**

(b) A concert has a loudness of 85 dB. Find $\frac{I}{I_0}$. **[2]**

9. 5 marks · Paper 1 style · calculator allowed

A piece of industrial machinery depreciates in value according to the model $V(t) = 45\,000e^{-0.15t}$, where V is the value in euros and t is the time in years since it was purchased.

1. Calculate the value of the machinery exactly 4 years after purchase. Round your answer to the nearest euro.
2. Using your GDC, find the exact number of years it will take for the machinery's value to halve.

10. **5 marks · Paper 1 style · calculator allowed**

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function $P(t) = Ae^{kt}$, where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment, and A and k are constants. The area covered by mould is $112\ \text{mm}^2$ at the start of the experiment and $360\ \text{mm}^2$ after 5 days.

1. Write down the value of A.
2. Using your GDC's solver, find the value of k, giving your answer correct to 3 significant figures.
3. Estimate the area covered by the mould after 8 days.

Answers and worked solutions

Question 1

2 marks

Answer: $x = 2.72$

$x = \log_5 80$ or $\dfrac{\log 80}{\log 5}$, or GDC **(M1)**

$x = 2.72$ **A1**

Question 2

2 marks

Answer: $x = 4.64$

$\log_{10} x = \dfrac{2}{3}$, so $x = 10^{2/3}$ **(M1)**

$x = 4.64$ **A1**

Question 3

5 marks

Answer: 20.1

Set up: $20 + 75e^{-0.08t} = 35$ **M1**

Isolate the exponential:

$75e^{-0.08t} = 15 \implies e^{-0.08t} = 0.2$ **A1**

Take natural logs: $-0.08t = \ln 0.2$ **M1**

$t = \frac{\ln 0.2}{-0.08}$ **A1**

$t = 20.1$ minutes (3 s.f.) **A1**

Question 4

4 marks

Answer: (a) 3; (b) 4; (c) 7

1. **CG50:** Press the **log** button. Enter 1000. EXE.
   $\log_{10} 1000 = 3$ **A1**
2. **CG50:** Press the **ln** button. Press **SHIFT** $\rightarrow$ $\ln$ (for e^x). Enter 4. EXE.
   $\ln(e^4) = 4$ **A1**
3. **CG50:** Press **SHIFT** $\rightarrow$ $\ln$ (for e^x). Press **ln** button, enter 7. EXE.
   e^x and $\ln x$ are inverse functions, so they cancel **(M1)**
   $e^{\ln 7} = 7$ **A1**

Question 5

4 marks

Answer: (a) 2.73; (b) 1.45

1. **CG50: MENU** $\rightarrow$ **Equation** $\rightarrow$ **Solver** $(F3)$.
   Enter $3^x = 20$ (use **SHIFT** $\rightarrow$ **.** for $=$). Press **SOLV** $(F6)$.
   $x = 2.73$ $(2.7268\ldots)$.
   (M1)A1
2. **CG50: MENU** $\rightarrow$ **Equation** $\rightarrow$ **Solver** $(F3)$.
   Enter $e^{2x} = 18$. Press **SOLV** $(F6)$.
   $x = 1.45$ $(1.4451\ldots)$.
   (M1)A1

Question 6

4 marks

Answer: (a) 4.49; (b) 6.31×10^{-7} moles per litre

(a)

$-\log_{10}(3.2 \times 10^{-5})$ **(M1)**

4.49 **A1**

(b)

$10^{-6.2}$ **(M1)**

6.31×10^{-7} **A1**

Question 7

4 marks

Answer: (a) 3410 (3406); (b) 19.8 years

(a)

$2400e^{0.35}$ **(M1)**

3406 $(3405.8\ldots)$ **A1**

(b)

$e^{0.035t} = 2$ **(M1)**

$t = 19.8$ years **A1**

Question 8

4 marks

Answer: (a) 30 dB; (b) 3.16×10^8

(a)

$10 \log_{10} 1000$ **(M1)**

30 dB **A1**

(b)

$\dfrac{I}{I_0} = 10^{8.5}$ **(M1)**

3.16×10^8 **A1**

Question 9

5 marks

Answer: (a) 24697; (b) 4.62

1. **CG50:** In Run-Matrix, type $45000 \times e^{-0.15\times 4}$ **(M1)**
   $V(4) = 24696.523\ldots \implies$ €$24\,697$ **A1**
2. Half the value $=$ €$22\,500$, so solve $22500 = 45000e^{-0.15t}$ **(M1)**
   CG50: MENU $\rightarrow$ **Equation** $\rightarrow$ **Solver** $(F3)$. Enter $22500 = 45000 \times e^{-0.15x}$. Press **SOLV** $(F6)$ **(M1)**
   $t = 4.62$ years $(4.62098\ldots)$ **A1**

Question 10

5 marks

Answer: (a) 112; (b) 0.234; (c) 725

1. Initial value at $t = 0$ is 112. Therefore, $A = 112$ **A1**
2. **CG50: MENU** $\rightarrow$ **Equation** $\rightarrow$ **Solver** $(F3)$.
   Enter $360 = 112 \times e^{5x}$.
   Press **SOLV** $(F6)$ **(M1)**
   $k = 0.234$ $(0.23352\ldots)$ **A1**
3. Substitute $t = 8$:
   $P(8) = 112 \times e^{0.23352\cdots\times 8}$ **(M1)**
   $= 725\ \text{mm}^2$ $(725.36\ldots)$ **A1**

Every solution is also on ibmathrevision.com/slai/logarithms-and-exponentials-exam-practice

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