IB MATHS AI SL · UNIT 5

Limits, Derivatives & Rates of Change: exam practice

10 questions · 39 marks · worked solutions and markschemes

Name _____ Date _____ Score / 39

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/limits-derivatives-and-rates-of-change-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

The cost, C euros, of producing x items is $C(x) = 0.1x^2 + 5x + 200$.

Find $C'(50)$ and explain what it means. **[3]**

2. **5 marks · Paper 2 style · calculator allowed**

Let $f(x) = 2x^3 - 9x^2 + 12x$.

Find the coordinates of the local maximum and local minimum points. **[5]**

3. **4 marks · Paper 2 style · calculator allowed**

Let $f(x) = x^4 - 4x^2 + x$.

Use your calculator to find the coordinates of the two local minimum points. **[4]**

4. **3 marks · Paper 1 style · calculator allowed**

The points $P(2, 4)$ and $Q(2+h,\ (2+h)^2)$ lie on the curve $y = x^2$.

(a) Find the gradient of the chord $[PQ]$ when $h = 0.1$. **[2]**

(b) Write down the value the gradient of $[PQ]$ approaches as h approaches 0. **[1]**

5. **4 marks · Paper 1 style · calculator allowed**

The distance travelled by a cyclist, s metres, t seconds after starting is $s = 4t^2 + 3t$.

(a) Find the speed of the cyclist when $t = 2$. **[2]**

(b) Find the average speed over the first 2 seconds. **[2]**

6. 3 marks · Paper 1 style · calculator allowed

The volume of water in a tank, V litres, t minutes after a tap is opened is $V = 200 - 0.3t^2$.

(a) Find $\frac{dV}{dt}$. **[1]**

(b) Find the value of $\frac{dV}{dt}$ when $t = 5$ and interpret it in context. **[2]**

7. 3 marks · Paper 1 style · calculator allowed

The table shows values of $f(x) = x^3$ near $x = 1$.

x	1	1.01	1.001
$f(x)$	1	1.030301	1.003003001

(a) Find the gradient of the chord from $x = 1$ to $x = 1.01$. **[1]**

(b) Find the gradient of the chord from $x = 1$ to $x = 1.001$. **[1]**

(c) Hence write down the gradient of the curve at $x = 1$. **[1]**

8. 4 marks · Paper 1 style · calculator allowed

Let $f(x) = x^3 - 12x$.

(a) Find $f'(x)$. **[1]**

(b) Find the values of x for which f is increasing. **[3]**

9. **5 marks · Paper 1 style · calculator allowed**

The population of a town, P thousand, t years after 2020 is modelled by $P(t) = 50 + 4t - 0.2t^2$.

(a) Find the rate of change of the population when $t = 5$. **[2]**

(b) Find the year in which the population is greatest, and the greatest population. **[3]**

10. **5 marks · Paper 2 style · calculator allowed**

Let $f(x) = \dfrac{3}{x} + x^2$, for $x > 0$.

(a) Find $f'(x)$. **[2]**

(b) Find the coordinates of the local minimum point. **[3]**

Answers and worked solutions

Question 1

3 marks

Answer: 15: about €15 per extra item

$C'(x) = 0.2x + 5$ (M1)

15 A1

each extra item costs about €15 when 50 are being made R1

Question 2

5 marks

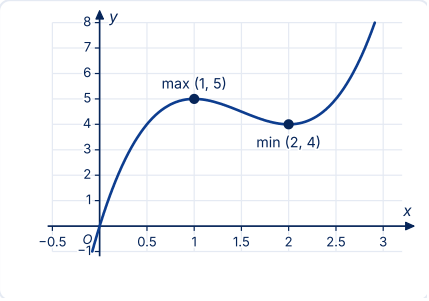
Answer: maximum $(1, 5)$, minimum $(2, 4)$

$f'(x) = 6x^2 - 18x + 12$ A1

$6(x - 1)(x - 2) = 0$ M1

local maximum $(1, 5)$ A1A1

local minimum $(2, 4)$ A1



Maximum (1, 5); minimum (2, 4)

Question 3

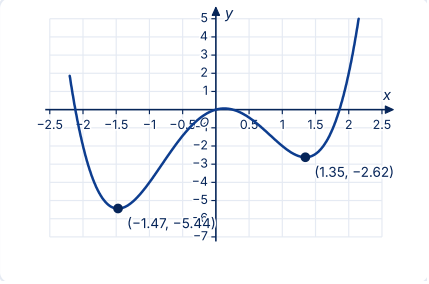
4 marks

Answer: $(-1.47,\ -5.44)$ and $(1.35,\ -2.62)$

$f'(x) = 4x^3 - 8x + 1 = 0$ or graph (M1)

$(-1.47,\ -5.44)$ A1A1

$(1.35,\ -2.62)$ A1



Two local minima

Question 4

3 marks

Answer: (a) 4.1; (b) 4

(a)

$\frac{2.1^2 - 4}{0.1}$ (M1)

4.1 A1

(b)

4 A1

Question 5

4 marks

Answer: (a) $19\ ms^{-1}$; (b) $11\ ms^{-1}$

(a)

$\frac{ds}{dt} = 8t + 3$ (M1)

$19\ ms^{-1}$ A1

(b)

$\frac{22}{2}$ (M1)

$11\ ms^{-1}$ A1

Question 6

3 marks

Answer: (a) $-0.6t$; (b) -3: the volume falls at 3 litres per minute

(a)

$-0.6t$ A1

(b)

-3 A1

the volume is falling at 3 litres per minute R1

Question 7

3 marks

Answer: (a) 3.0301; (b) 3.003001; (c) 3

(a)

3.0301 A1

(b)

3.003001 A1

(c)

3 A1

Question 8

4 marks

Answer: (a) $3x^2 - 12$; (b) $x < -2$ or $x > 2$

(a)

$3x^2 - 12$ A1

(b)

$3x^2 - 12 > 0$ (M1)

critical values ± 2 (A1)

$x < -2$ or $x > 2$ A1

Question 9

5 marks

Answer: (a) 2 thousand per year; (b) 2030 ($t = 10$), 70 thousand

(a)

$P'(t) = 4 - 0.4t$ (M1)

2 thousand per year A1

(b)

$4 - 0.4t = 0$ (M1)

$t = 10$, in 2030 A1

70 thousand A1

Question 10

5 marks

Answer: (a) $-\frac{3}{x^2} + 2x$; (b) $(1.14,\ 3.93)$

(a)

$-3x^{-2} + 2x$ A1A1

(b)

$2x = \frac{3}{x^2}$, $x^3 = 1.5$ (M1)

$x = 1.14$ A1

$(1.14,\ 3.93)$ A1

Every solution is also on ibmathrevision.com/slai/limits-derivatives-and-rates-of-change-exam-practice
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