Exponential Modelling: exam practice

10 questions · 46 marks · worked solutions and markschemes

Name _____ Date _____ Score / 46

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/exponential-modelling-exam-practice

1. **3 marks · Paper 2 style · calculator allowed**

Account A starts with €2000 and grows by 5% a year. Account B starts with €2500 and grows by 3% a year.

Find the number of years after which account A is first worth more than account B. **[3]**

2. **4 marks · Paper 2 style · calculator allowed**

The amount of a drug in the bloodstream, M mg, t hours after a dose is $M(t) = 400(0.8)^t$.

(a) Find the amount after 3 hours. **[2]**

(b) Find the time when the amount first falls below 50 mg. **[2]**

3. **4 marks · Paper 2 style · calculator allowed**

A radioactive substance has an initial mass of $80\ \text{g}$. It decays at a continuous rate such that its mass $M(t)$ after t days is given by $M(t) = 80(0.88)^t$.

1. Calculate the mass remaining after exactly 10 days.
2. Find the exact half-life of the substance (the time taken for the mass to halve).

4. **4 marks · Paper 2 style · calculator allowed**

The mass of a radioactive substance is $m = 80e^{-kt}$ grams after t days. Its half-life is 6 days.

(a) Find the value of k. **[2]**

(b) Find the mass after 15 days. **[2]**

5. **5 marks · Paper 2 style · calculator allowed**

The population of a species of fish in a lake is growing according to the exponential model $P(t) = 1500(1.06)^t$, where t is the number of years since observation began.

1. Find the expected fish population after 5 years.
2. Using algebraic logarithms or your Graphic Display Calculator, calculate the exact time it will take for the initial population to double. Give your answer correct to 3 significant figures.

6. **5 marks · Paper 1 style · calculator allowed**

A population is modelled by $P(t) = a \times b^t$, where t is in years. $P(0) = 500$ and $P(2) = 720$.

(a) Find a and b. **[3]**

(b) Find $P(5)$. **[2]**

7. **4 marks · Paper 2 style · calculator allowed**

The amount of a particular medicine in a patient's bloodstream, M in milligrams, t hours after administration is modelled by $M(t) = 250(0.85)^t$.

1. Write down the initial amount of medicine administered.
2. Calculate the exact half-life of the medicine in the bloodstream.

8. **4 marks · Paper 2 style · calculator allowed**

The volume of water in a tank, V litres, t minutes after a tap is opened is $V(t) = 60 - 45e^{-0.2t}$.

(a) Write down the volume when the tap is opened. **[1]**

(b) Write down the volume the tank approaches in the long term. **[1]**

(c) Find the time for the volume to reach 50 litres. **[2]**

9. **5 marks · Paper 2 style · calculator allowed**

The value of a vintage motorbike, $V(t)$ in euros, t years after the year 2015 is modelled by the function $V(t) = 9600 - 2400(1.08)^t$.

1. Calculate the expected value of the motorbike in the year 2018.
2. The model predicts that eventually the motorbike will become worthless. Using your Graphic Display Calculator, find the year in which the value of the motorbike drops below zero.

10. **8 marks · Paper 2 style · calculator allowed**

A heated metal object is placed in a cooling bath. Its temperature, T in $°\text{C}$, after t minutes is modelled by $T(t) = 18 + 105e^{-0.12t}$.

1. Write down the ambient temperature of the cooling bath.
2. Find the exact time it takes for the object to reach $40°\text{C}$.
3. A second object's temperature is modelled by $S(t) = 15 + 120e^{-0.15t}$. Using your GDC, find the exact time when both objects are at the identical temperature.

Answers and worked solutions

Question 1

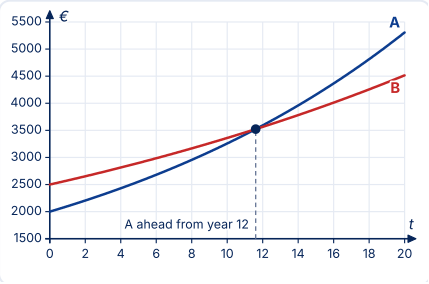
3 marks

Answer: 12 years

$2000(1.05)^t > 2500(1.03)^t$ (M1)

$t > 11.6$ (A1)

12 years A1



The curves cross between $t = 11$ and $t = 12$

Question 2

4 marks

Answer: (a) 205 mg; (b) 9.32 hours

(a)

400×0.8^3 (M1)

205 mg (204.8) A1

(b)

$0.8^t < 0.125$ (M1)

9.32 hours A1

Question 3

4 marks

Answer: (a) 22.3; (b) 5.42

1. Substitute $t = 10$:
   $M(10) = 80(0.88)^{10}$ M1
   ≈ 22.3 g A1
2. Set
   $40 = 80(0.88)^t \implies 0.88^t = 0.5$
   M1
   Take logs:
   $t = \frac{\ln(0.5)}{\ln(0.88)} \approx 5.42$ days
   A1

Question 4

4 marks

Answer: (a) $k = 0.116$; (b) 14.1 g

(a)

$e^{-6k} = 0.5$ (M1)

$k = 0.116$ A1

(b)

$80e^{-15k}$ (M1)

14.1 g A1

Question 5

5 marks

Answer: (a) 2007; (b) 11.9

1. Substitute $t = 5$:
   $P(5) = 1500(1.06)^5$ M1
   ≈ 2007 fish A1
2. Set
   $1500(1.06)^t = 3000 \implies 1.06^t = 2$
   M1
   Take logs: $t = \frac{\ln(2)}{\ln(1.06)}$ M1
   $t \approx 11.9$ years A1

Question 6

5 marks

Answer: (a) $a = 500,\ b = 1.2$; (b) 1240

(a)

$a = 500$ A1

$500b^2 = 720$ (M1)

$b = 1.2$ A1

(b)

500×1.2^5 (M1)

1240 (1244.16) A1

Question 7

4 marks

Answer: (a) 250; (b) 4.27

1. $M(0) = 250$ mg A1
2. Set the amount equal to half the initial amount:
   $125 = 250(0.85)^t$ M1
   Divide by 250: $0.85^t = 0.5$
   M1
   Take logs:
   $t = \frac{\ln(0.5)}{\ln(0.85)} \approx 4.27$ hours
   A1

Question 8

4 marks

Answer: (a) 15 litres; (b) 60 litres; (c) 7.52 minutes

(a)

15 litres A1

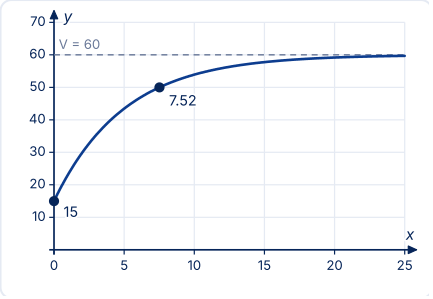
(b)

60 litres A1

(c)

$45e^{-0.2t} = 10$ (M1)

7.52 minutes A1



Starts at 15; tends to 60

Question 9

5 marks

Answer: (a) 6576.69; (b) 2033

1. For the year 2018, $t = 3$.
   $V(3) = 9600 - 2400(1.08)^3 = 9600 - 3023.3$
   M1
   $= €6576.69$ A1

2. Set
$9600 - 2400(1.08)^t = 0$
M1
Rearrange:
$1.08^t = \frac{9600}{2400} = 4$
Use GDC or logs:
$t = \frac{\ln(4)}{\ln(1.08)} \approx 18.01$ **A1**
$V(t) < 0$ for $t > 18.01$, just after the start of $2015 + 18$, so the value drops below zero in the year 2033. **A1**

Every solution is also on ibmathrevision.com/slai/exponential-modelling-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.

Question 10 — 8 marks

Answer: (a) 18; (b) 13.0; (c) 3.10

1. As $t \to \infty$, $e^{-0.12t} \to 0$, so $T \to 18$. Ambient temperature is $18°\text{C}$. **A1**
2. Set $T(t) = 40$:
$40 = 18 + 105e^{-0.12t}$ **M1**
Subtract 18 and divide by 105: $e^{-0.12t} = \frac{22}{105}$
Take natural logs:
$-0.12t = \ln\left(\frac{22}{105}\right)$ **M1**
$t = \frac{1}{0.12}\ln\left(\frac{105}{22}\right) \approx 13.0 \text{ minutes}$
A1
3. Set
$18 + 105e^{-0.12t} = 15 + 120e^{-0.15t}$
M1
Graph both functions on the GDC and find the intersection (G-Solv $\to$ ISCT). **(M1)**
$t \approx 3.10 \text{ minutes}$
$(3.0977\ldots)$ **A1**
Both objects are then at $T \approx 90.4°\text{C}$. **A1**