Direct/Inverse Variation and Cubic Models: exam practice

10 questions · 39 marks · worked solutions and markschemes

Name _____ Date _____ Score / 39

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/direct-inverse-variation-and-cubic-models-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

The time, T days, needed to finish a job varies inversely with the number of workers, w. Six workers need 15 days.

Find how many days ten workers need. **[3]**

2. **3 marks · Paper 1 style · calculator allowed**

The cost, C euros, of a spherical chocolate varies directly as the cube of its radius, r cm. A chocolate of radius 2 cm costs €1.60.

Find the cost of a chocolate of radius 3 cm. **[3]**

3. **3 marks · Paper 1 style · calculator allowed**

The variable y varies directly with the cube of x. When $x = 3$, $y = 54$.

1. Find the constant of proportionality, k, and write down the equation linking y and x.
2. Calculate the value of y when $x = 5$.

4. **3 marks · Paper 1 style · calculator allowed**

y is inversely proportional to x, and $y = 12$ when $x = 4$.

(a) Find y when $x = 16$. **[2]**

(b) State what happens to y when x is doubled. **[1]**

5. **5 marks · Paper 1 style · calculator allowed**

y varies directly as the square of x. When $x = 3$, $y = 45$.

(a) Find an equation for y in terms of x. **[2]**

(b) Find y when $x = 7$. **[1]**

(c) Find the positive value of x when $y = 320$. **[2]**

6. **3 marks · Paper 2 style · calculator allowed**

A power model $y = ax^b$ passes through the points $(1, 3)$ and $(2, 24)$. Find a and b.

7. **4 marks · Paper 1 style · calculator allowed**

The force of attraction, F Newtons, between two magnets varies inversely with the square of the distance, d cm, between them. When $d = 4$, $F = 150$.

1. Find the equation linking F and d.
2. Calculate the force when the distance is increased to $10\ \text{cm}$.

8. **5 marks · Paper 2 style · calculator allowed**

A cubic model $f(x) = ax^3 + bx^2 + cx + d$ passes through the points $(0, 2)$, $(1, 5)$, $(2, 16)$ and $(3, 41)$.

(a) Write down the value of d. **[1]**

(b) Find the values of a, b and c. **[3]**

(c) Use the model to find $f(4)$. **[1]**

9. **5 marks · Paper 1 style · calculator allowed**

The intensity of light, I, from a lamp varies inversely as the square of the distance, d metres, from the lamp. When $d = 2$, $I = 200$.

(a) Find an equation for I in terms of d. **[2]**

(b) Find I when $d = 5$. **[1]**

(c) Find the distance at which $I = 50$. **[2]**

10. **5 marks · Paper 2 style · calculator allowed**

The number of items, P, produced by a machine t hours after it is switched on is modelled by $P(t) = -0.5t^3 + 6t^2 + 10t$, for $0 \leq t \leq 12$.

(a) Find the number of items produced after 4 hours. **[2]**

(b) Use your calculator to find the maximum value of P and the time at which it occurs. **[3]**

Answers and worked solutions

Question 1

3 marks

Answer: 9 days

$T = \frac{k}{w}$, $k = 90$ (M1)(A1)

9 days A1

Question 2

3 marks

Answer: €5.40

$1.6 = k \times 8$, $k = 0.2$ (M1)(A1)

€5.40 A1

Question 3

3 marks

Answer: (a) $y =) \ 2x^3$; (b) 250

1. Write $y = kx^3$ and substitute $x = 3,\ y = 54$:
   $54 = k(3^3) = 27k$ M1
   $k = 2$, so $y = 2x^3$ A1
2. Substitute $x = 5$:
   $y = 2(5^3) = 2(125) = 250$
   A1

Question 4

3 marks

Answer: (a) 3

(a)

$k = 48$ (M1)

$y = 3$ A1

(b)

y is halved A1

Question 5

5 marks

Answer: (a) $y = 5x^2$; (b) 245; (c) 8

(a)

$45 = k \times 9$ (M1)

$y = 5x^2$ A1

(b)

245 A1

(c)

$x^2 = 64$ (M1)

$x = 8$ A1

Question 6

3 marks

Answer: $a = 3$; $b = 3$

$a \times 1^b = 3$, so $a = 3$ A1

$3 \times 2^b = 24$, so $2^b = 8$ (M1)

$b = 3$ A1

Question 7

4 marks

Answer: (a) $F =) \ 2400/d^2$; (b) 24

1. Write $F = \frac{k}{d^2}$ and substitute $d = 4,\ F = 150$:
   $150 = \frac{k}{16}$ M1
   $k = 150 \times 16 = 2400$, so $F = \frac{2400}{d^2}$ A1
2. Substitute $d = 10$:
   $F = \frac{2400}{10^2} = \frac{2400}{100}$ M1
   $F = 24\ \text{N}$ A1

Question 8

5 marks

Answer: (a) 2; (b) $a = 1,\ b = 1,\ c = 1$; (c) 86

(a)

2 A1

(b)

$a + b + c = 3$,
$8a + 4b + 2c = 14$,
$27a + 9b + 3c = 39$ (M1)(A1)

$a = b = c = 1$ A1

(c)

86 A1

Question 9

5 marks

Answer: (a) $I = \frac{800}{d^2}$; (b) 32; (c) $4\ m$

(a)

$200 = \frac{k}{4}$ (M1)

$I = \frac{800}{d^2}$ A1

(b)

32 A1

(c)

$d^2 = 16$ (M1)

$d = 4\ m$ A1

Question 10

5 marks

Answer: (a) 104; (b) $t = 8.76$ hours, $P = 212$

(a)

$-32 + 96 + 40$ (M1)

104 A1

(b)

GDC maximum (M1)

$t = 8.76$ A1

$P = 212$ A1

Every solution is also on ibmathrevision.com/slai/direct-inverse-variation-and-cubic-models-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.