Applications of Functions: exam practice

10 questions · 50 marks · worked solutions and markschemes

Name _____ Date _____ Score / 50

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/applications-of-functions-exam-practice

1. **5 marks · Paper 1 style · calculator allowed**

The value of a specific industrial machine, V, in euros, can be modelled as a linear function of time, t, in years since it was purchased. The model is given by $V(t) = 12\,500 - 1100t$.

1. Write down the initial purchase price of the machine.
2. Calculate the value of the machine exactly 4 years after purchase.
3. Find the exact time, t, when the machine loses all of its value ($V = 0$).

2. **5 marks · Paper 1 style · calculator allowed**

The value of a car, V, in euros, t years after it was purchased is modelled by the exponential decay function $V(t) = 24\,000(0.82)^t$.

1. Write down the initial purchase price of the car.
2. Calculate the value of the car exactly 5 years after it was purchased.
3. Using your GDC, find the exact number of years it takes for the car's value to drop to €$12\,000$.

3. **5 marks · Paper 1 style · calculator allowed**

A taxi fare, F euros, for a trip of d km is $F = 3.5 + 1.8d$.

(a) Interpret the value 1.8 in this context. **[1]**

(b) Find the fare for a 12 km trip. **[2]**

(c) Find the greatest distance that can be travelled for €40. **[2]**

4. **4 marks · Paper 2 style · calculator allowed**

The value of a motorbike, V euros, t years after it is bought is $V(t) = 25\,000(0.85)^t$.

(a) Find the value after 3 years, to the nearest euro. **[2]**

(b) Find the number of complete years until the value is first below €$10\,000$. **[2]**

5. **4 marks · Paper 1 style · calculator allowed**

The number of downloads of an app, N, t days after its launch is modelled by $N = 500\log_{10}(t+1)$.

(a) Find N when $t = 99$. **[2]**

(b) Find the value of t when $N = 1500$. **[2]**

6. **5 marks · Paper 1 style · calculator allowed**

The volume of water in a tank, V in litres, t minutes after a pump is switched on is modelled by the quadratic function $V(t) = -2t^2 + 40t + 100$.

1. Find the volume of water in the tank when the pump is first switched on.
2. Find the time t at which the volume of water is at its maximum.
3. Calculate this maximum volume.

7. **5 marks · Paper 2 style · calculator allowed**

The height of a plant, h cm, t weeks after it is planted is modelled by $h(t) = 3 + 4\ln(t + 1)$.

(a) Write down the height of the plant when it is planted. **[1]**

(b) Find the height after 5 weeks. **[2]**

(c) Find the time taken for the plant to reach 12 cm. **[2]**

8. **6 marks · Paper 1 style · calculator allowed**

The height of a baseball after it is hit by a bat is modelled by the quadratic function $h(t) = -4.8t^2 + 21t + 1.2$, where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

1. Write down the height of the ball above the ground at the exact instant it is hit.
2. Find the maximum height reached by the baseball.
3. Use your GDC to find the value of t when the ball hits the ground.

9. **5 marks · Paper 2 style · calculator allowed**

A company makes x items a week. The weekly cost is $C(x) = 0.02x^2 + 3x + 500$ euros and each item sells for €12.

(a) Show that the weekly profit is $P(x) = -0.02x^2 + 9x - 500$. **[1]**

(b) Find the two values of x for which the company breaks even. **[2]**

(c) Find the maximum weekly profit. **[2]**

10. **6 marks · Paper 1 style · calculator allowed**

A biologist uses the following quadratic model to estimate the growth rate of a specific algae culture, y, at varying temperatures, x (in degrees Celsius):

$$y = -0.45x^2 + 3.2x - 1.7$$

1. Using your GDC's maximum tool, find the temperature x that produces the maximum growth rate, and state this maximum rate.
2. Using your GDC, find the two temperatures where the growth rate y is exactly zero.

Answers and worked solutions

Question 1

5 marks

Answer: (a) 12500; (b) 8100; (c) 11.4

1. Substitute $t = 0$:
   $V(0) = 12\,500 - 1100(0) = €12\,500$. **A1**
2. Substitute $t = 4$:
   $V(4) = 12\,500 - 1100(4) = 12\,500 - 4400$. **(M1)**
   $V(4) = €8\,100$. **A1**
3. Set $V(t) = 0$:
   $0 = 12\,500 - 1100t$. **(M1)**
   Solve:
   $t = \frac{12\,500}{1100} = \frac{125}{11} \approx 11.4$ years ($11.3636\ldots$). **A1**

Question 2

5 marks

Answer: (a) 24000; (b) 8900; (c) 3.49

1. Substitute $t = 0$:
   $V(0) = 24\,000(0.82)^0 = €24\,000$. **A1**
2. Substitute $t = 5$:
   $V(5) = 24\,000(0.82)^5$. **(M1)**
   $V(5) = 8897.76\ldots \approx €8900$ (accept $€8897.76$). **A1**
3. Set $12\,000 = 24\,000(0.82)^t$ and solve with GDC (**CG50: Equation → Solver**) or logs:
   $t = \frac{\ln(0.5)}{\ln(0.82)}$. **(M1)**
   $t \approx 3.49$ years ($3.4927\ldots$). **A1**

Question 3

5 marks

Answer: (b) $€25.10$; (c) 20.3 km

(a)

the cost per kilometre, $€1.80$ **A1**

(b)

$3.5 + 1.8 \times 12$ **(M1)**

$€25.10$ **A1**

(c)

$3.5 + 1.8d = 40$ **(M1)**

20.3 km **A1**

Question 4

4 marks

Answer: (a) $€15\,353$; (b) 6 years

(a)

$25\,000 \times 0.85^3$ **(M1)**

$€15\,353$ **A1**

(b)

$0.85^t < 0.4$, $t > 5.64$ **(M1)**

6 years **A1**

Question 5

4 marks

Answer: (a) 1000; (b) 999

(a)

$500 \log_{10} 100$ **(M1)**

1000 **A1**

(b)

$\log_{10}(t + 1) = 3$ **(M1)**

$t = 999$ **A1**

Question 6

5 marks

Answer: (a) 100; (b) 10; (c) 300

1. $t = 0 \implies V(0) = 100$ Litres. **A1**
2. Max volume occurs at vertex
   $t = \frac{-b}{2a} = \frac{-40}{2(-2)} = 10$ min. **(M1)A1**
3. $V(10) = -2(10)^2 + 40(10) + 100 = -200 + 400 + 100 = 300$ Litres. **(M1)A1**

Question 7

5 marks

Answer: (a) 3 cm; (b) 10.2 cm; (c) 8.49 weeks

(a)

3 cm **A1**

(b)

$3 + 4 \ln 6$ **(M1)**

10.2 cm **A1**

(c)

$\ln(t + 1) = 2.25$ **(M1)**

8.49 weeks **A1**

Question 8

6 marks

Answer: (a) 1.2; (b) 24.2; (c) 4.43

1. Substitute $t = 0$:
   $h(0) = 1.2$ m. **A1**
2. Vertex: $t = \frac{-21}{2(-4.8)}$. **(M1)**
   $t = 2.1875$ s. **A1**
   Substitute:
   $h(2.1875) = -4.8(2.1875)^2 + 21(2.1875) + 1.2 \approx (24.168\ldots)$. **A1**
3. Set $h(t) = 0$:
   $-4.8t^2 + 21t + 1.2 = 0$ (**CG50: Equation → Polynomial**). **(M1)**
   $t \approx 4.43$ s ($4.431\ldots$); reject the negative root
   $t \approx -0.0564$. **A1**

Question 9

5 marks

Answer: (b) $x = 64.9$ and $x = 385$; (c) $€512.50$

(a)

$12x - (0.02x^2 + 3x + 500)$ **A1**

AG

(b)

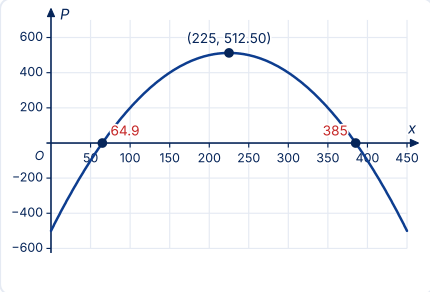
$P(x) = 0$ **(M1)**

$x = 64.9$ and $x = 385$ **A1**

(c)

$x = 225$ **(M1)**

$€512.50$ **A1**



Break even at 64.9 and 385; maximum €512.50 at $x = 225$

Question 10

6 marks

Answer: (a) temperature) $\mathbf{3.56}$; (a) maximum rate) $\mathbf{3.99}$; (b) $\mathbf{0.578, 6.53}$

1. **CG50: Graph $\rightarrow$ G-Solv $\rightarrow$ MAX** (or $x = \frac{-b}{2a} = \frac{-3.2}{2(-0.45)}$). **(M1)**
   $x \approx 3.56°\text{C}$ $(3.555\ldots)$. **A1**
   Maximum growth rate
   $y \approx 3.99$ $(3.988\ldots)$. **A1**
2. **CG50: Equation $\rightarrow$ Polynomial $\rightarrow$ Degree 2** (or **G-Solv $\rightarrow$ ROOT**) for $-0.45x^2 + 3.2x - 1.7 = 0$. **(M1)**
   $x_1 \approx 0.578°\text{C}$ $(0.5782\ldots)$. **A1**
   $x_2 \approx 6.53°\text{C}$ $(6.5328\ldots)$. **A1**

Every solution is also on ibmathrevision.com/slai/applications-of-functions-exam-practice
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