3D Coordinate Geometry: exam practice

10 questions · 40 marks · worked solutions and markschemes

Name _____ Date _____ Score / 40

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slai/3d-coordinate-geometry-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

The point $C(k, 2, 1)$ is 6 units from the origin.

Find the possible values of k. **[3]**

2. **3 marks · Paper 1 style · calculator allowed**

The points $P(-1, 3, 5)$ and $Q(5, 7, -1)$ represent the endpoints of a tunnel in a 3D grid.

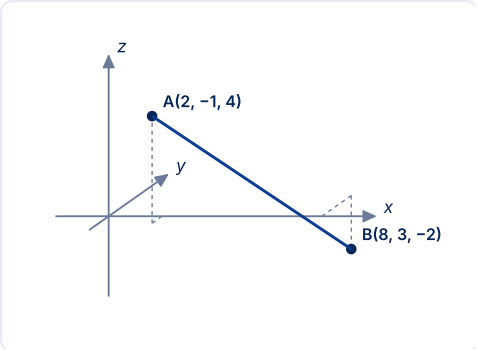
1. Find the exact coordinates of the midpoint of the tunnel.
2. Find the exact distance between point P and point Q.

3. **4 marks · Paper 1 style · calculator allowed**

The points $A(2, -1, 4)$ and $B(8, 3, -2)$ are shown on a three-dimensional grid.

(a) Find the coordinates of the midpoint of $[AB]$. **[2]**

(b) Find the distance AB. **[2]**



A(2, −1, 4) and B(8, 3, −2)

4. **4 marks · Paper 2 style · calculator allowed**

A rectangular room has dimensions: length $8\ \text{m}$, width $5\ \text{m}$, and height $4\ \text{m}$.

1. Calculate the exact length of the space diagonal (the straight line from one bottom corner to the opposite top corner).
2. Calculate the angle this space diagonal makes with the floor of the room.

5. **4 marks · Paper 1 style · calculator allowed**

Consider the 3D points $A(-2, 3, 5)$ and $B(4, 1, -1)$.

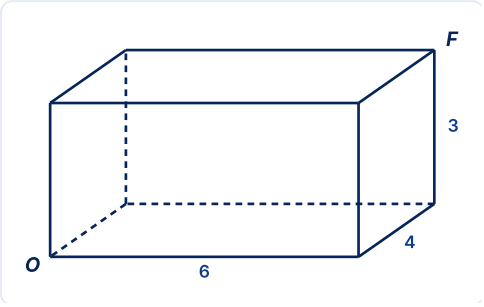
1. Find the exact coordinates of the midpoint of the line segment $[AB]$.
2. Find the exact distance between point A and point B.

6. **4 marks · Paper 2 style · calculator allowed**

A cuboid has one vertex at the origin O, edges along the positive axes, and the opposite vertex at $F(6, 4, 3)$.

(a) Find the length OF. **[2]**

(b) Find the angle between $[OF]$ and the base of the cuboid (the xy-plane). **[2]**



Cuboid from O to F(6, 4, 3)

7. **4 marks · Paper 1 style · calculator allowed**

$M(1, 4, -2)$ is the midpoint of $[PQ]$, where P is $(-3, 7, 0)$.

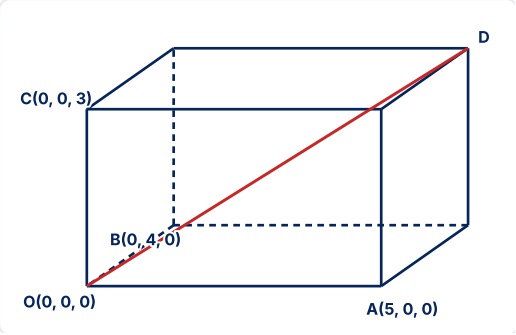
(a) Find the coordinates of Q. **[2]**

(b) Find the length of $[PQ]$. **[2]**

8. **4 marks · Paper 1 style · calculator allowed**

A rectangular cuboid sits on a 3D coordinate system. Its vertices include the origin $O(0, 0, 0)$, and the points $A(5, 0, 0)$, $B(0, 4, 0)$, and $C(0, 0, 3)$. Let point D be the vertex diagonally opposite to the origin.

1. State the coordinates of point D.
2. Find the exact length of the space diagonal OD.
3. Find the exact coordinates of the midpoint of OD.



Cuboid on the axes; diagonal OD

9. **5 marks · Paper 1 style · calculator allowed**

A right square-based pyramid has a base with side lengths of $12\ \text{cm}$ and a vertical height of $16\ \text{cm}$.

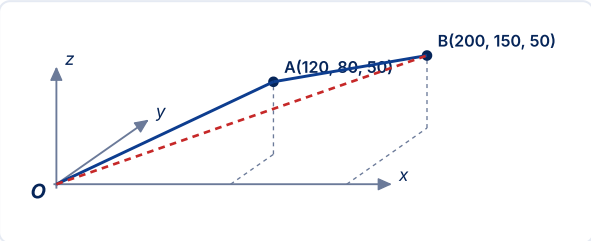
1. Calculate the length of the slant edge of the pyramid.
2. Find the angle between a sloping triangular face and the square base.

10. **5 marks · Paper 2 style · calculator allowed**

A drone takes off from $O(0, 0, 0)$ and flies in a straight line to $A(120, 80, 50)$, then in a straight line to $B(200, 150, 50)$. Distances are in metres.

(a) Find the total distance flown. **[3]**

(b) Find how much shorter a direct flight from O to B would be. **[2]**



Drone path O to A to B

Answers and worked solutions

Question 1 — 3 marks

Answer: $k = \pm 5.57$ $(\pm\sqrt{31})$

$k^2 + 4 + 1 = 36$ **M1**

$k^2 = 31$ **(A1)**

$k = \pm 5.57$ **A1**

Question 2 — 3 marks

Answer: (a) $(2, 5, 2)$; (b) $\sqrt{88}$

1. Midpoint
   $= \left(\frac{-1+5}{2}, \frac{3+7}{2}, \frac{5+(-1)}{2}\right) = (2, 5, 2)$
   A1
2. Distance
   $= \sqrt{(5-(-1))^2 + (7-3)^2 + (-1-5)^2}$
   M1
   $= \sqrt{6^2 + 4^2 + (-6)^2} = \sqrt{36+16+36} = \sqrt{88}$ $\left(= 2\sqrt{22}\right)$
   A1

Question 3 — 4 marks

Answer: (a) $(5, 1, 1)$; (b) 9.38 ($\sqrt{88}$)

(a)

$$\left(\frac{2+8}{2}, \frac{-1+3}{2}, \frac{4-2}{2}\right)$$

(M1)

$(5, 1, 1)$ **A1**

(b)

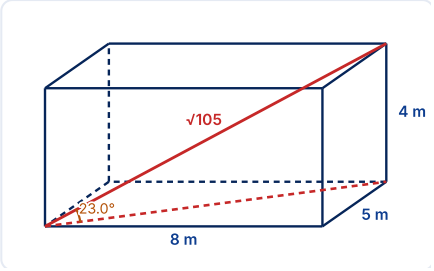
$\sqrt{6^2 + 4^2 + (-6)^2}$ **(M1)**

9.38 **A1**

Question 4 — 4 marks

Answer: (a) $\sqrt{105}$; (b) $23.0°$

1. Diagonal
   $= \sqrt{8^2 + 5^2 + 4^2}$ **M1**
   $= \sqrt{64 + 25 + 16} = \sqrt{105}$ m
   (or ≈ 10.2 m). **A1**
2. Base diagonal
   $= \sqrt{8^2 + 5^2} = \sqrt{89}$. **M1**
   $\tan\alpha = \frac{4}{\sqrt{89}} \implies \alpha = \tan^{-1}(4/\sqrt{89}) \approx 23.0°$
   . **A1**



Space diagonal $\sqrt{105}m$ at $23.0°$ to the floor

Question 5 — 4 marks

Answer: (a) $(1, 2, 2)$; (b) $\sqrt{76}$

1. Midpoint
   $= \left(\frac{-2+4}{2}, \frac{3+1}{2}, \frac{5+(-1)}{2}\right) = (1, 2, 2)$
   . **M1 A1**
2. Distance
   $= \sqrt{(4-(-2))^2 + (1-3)^2 + (-1-5)^2}$
   M1
   $= \sqrt{6^2 + (-2)^2 + (-6)^2} = \sqrt{36+4+36} = \sqrt{76}$
   . **A1**

Question 6 — 4 marks

Answer: (a) 7.81 $(\sqrt{61})$; (b) $22.6°$

(a)

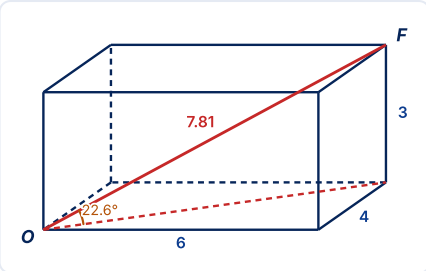
$\sqrt{36 + 16 + 9}$ **(M1)**

7.81 **A1**

(b)

$$\sin\theta = \frac{3}{\sqrt{61}}$$ **(M1)**

$22.6°$ **A1**



OF $= \sqrt{61} = 7.81$ at $22.6°$ to the base

Question 7 — 4 marks

Answer: (a) $(5, 1, -4)$; (b) 10.8

(a)

$Q = 2M - P$ **(M1)**

$(5, 1, -4)$ **A1**

(b)

$\sqrt{8^2 + 6^2 + 4^2}$ **(M1)**

10.8 **A1**

Question 8 — 4 marks

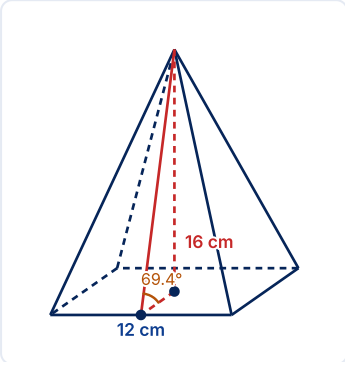
Answer: (a) $(5, 4, 3)$; (b) $5\sqrt{2}$; (c) $(2.5, 2, 1.5)$

1. D is diagonally opposite the origin: $D(5, 4, 3)$ **A1**
2. Use the 3D distance formula:
   $OD = \sqrt{5^2 + 4^2 + 3^2}$
   M1
   $= \sqrt{25 + 16 + 9} = \sqrt{50} = 5\sqrt{2}$
   A1
3. Midpoint
   $= \left(\frac{0+5}{2}, \frac{0+4}{2}, \frac{0+3}{2}\right) = \left(\frac{5}{2}, 2, \frac{3}{2}\right)$
   A1

Question 9 — 5 marks

Answer: (a) 18.1; (b) $69.4°$

1. Diagonal of the base: $\sqrt{12^2+12^2}=\sqrt{288}=12\sqrt{2}$ **M1**
   Distance from the centre of the base to a corner: $\frac{12\sqrt{2}}{2}=6\sqrt{2}$
   Use Pythagoras with the vertical height: $l=\sqrt{16^2+(6\sqrt{2})^2}$ **M1**
   $l=\sqrt{256+72}=\sqrt{328}\approx 18.1$ cm **A1**
2. Use the right-angled triangle with vertical height 16 and horizontal distance 6 (centre of base to midpoint of an edge) **M1**
   $\tan\theta=\frac{16}{6}\implies\theta=\tan^{-1}\left(\frac{8}{3}\right)\approx 69.4°$ **A1**



Face angle at M: $69.4°$

Question 10

5 marks

Answer: (a) $259\ m$; (b) $3.99\ m$

(a)

$OA=\sqrt{120^2+80^2+50^2}=152.6\ldots$ **(M1)**

$AB=\sqrt{80^2+70^2}=106.3\ldots$ **(A1)**

$259\ m$ **A1**

(b)

$OB=\sqrt{200^2+150^2+50^2}=254.9\ldots$ **(M1)**

$3.99\ m$ **A1**

Every solution is also on ibmathrevision.com/slai/3d-coordinate-geometry-exam-practice
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