IB MATHS AA SL · UNIT 3

Trigonometric Identities: exam practice

10 questions · 51 marks · worked solutions and markschemes

Name _____ Date _____ Score / 51

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/trigonometric-identities-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. **[3]**

2. **4 marks · Paper 1 style · no calculator**

Show that $\dfrac{1 - \cos 2x}{\sin 2x} = \tan x$, for $\sin 2x \neq 0$. **[4]**

3. **5 marks · Paper 1 style · no calculator**

Given that $\sin\theta = \frac{4}{5}$ and θ is an obtuse angle ($\frac{\pi}{2} < \theta < \pi$), find the exact value of $\sin(2\theta)$.

4. 6 marks · Paper 1 style · no calculator

Solve $2\sin^2 x = 1 + \cos x$ for $0 \leq x \leq 2\pi$. **[6]**

5. 5 marks · Paper 1 style · no calculator

Solve $\sin 2x = \tan x$ for $0 < x < \pi$. **[5]**

6. 6 marks · Paper 1 style · no calculator

Solve $\cos 2x = \cos x$ for $0 \leq x \leq 2\pi$. **[6]**

7. 5 marks · Paper 1 style · no calculator

Given that $\sin\theta = \frac{1}{3}$ and $0 < \theta < \frac{\pi}{2}$,

(a) find the exact value of $\cos 2\theta$; **[2]**

(b) find the exact value of $\sin 2\theta$. **[3]**

8. 5 marks · Paper 1 style · no calculator

(a) Show that $\sin^4 x - \cos^4 x = -\cos 2x$. **[3]**

(b) Hence find the exact value of $\sin^4 \frac{\pi}{12} - \cos^4 \frac{\pi}{12}$. **[2]**

9. 6 marks · Paper 1 style · no calculator

Given that $\cos x = \frac{1}{4}$ and $0 < x < \frac{\pi}{2}$, find the exact values of:

1. $\cos(2x)$
2. $\sin(2x)$

10. **6 marks · Paper 1 style · no calculator**

(a) Show that the equation $\cos 2x + 3\sin x + 1 = 0$ can be written as $2\sin^2 x - 3\sin x - 2 = 0$. **[2]**

(b) Hence solve $\cos 2x + 3\sin x + 1 = 0$ for $0 \leq x \leq 2\pi$. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: a proof: the worked solution above is the full answer

$\sin^2 x + 2\sin x \cos x + \cos^2 x$ M1

$\sin^2 x + \cos^2 x = 1$ A1

$2\sin x \cos x = \sin 2x$, so $1 + \sin 2x$ A1 AG

Question 2 — 4 marks

Answer: a proof: the worked solution above is the full answer

$1 - \cos 2x = 2\sin^2 x$ A1

$\sin 2x = 2\sin x \cos x$ A1

$\dfrac{2\sin^2 x}{2\sin x \cos x}$ M1

$= \dfrac{\sin x}{\cos x} = \tan x$ A1 AG

Question 3 — 5 marks

Answer: $-24/25$

Use $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{16}{25} = \frac{9}{25}$ M1

θ is obtuse, so $\cos\theta < 0$ R1

$\cos\theta = -\frac{3}{5}$ A1

Use $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$ M1

$\sin 2\theta = -\frac{24}{25}$ A1

Question 4 — 6 marks

Answer: $x = \dfrac{\pi}{3},\ \pi,\ \dfrac{5\pi}{3}$

$2(1 - \cos^2 x) = 1 + \cos x$ M1

$2\cos^2 x + \cos x - 1 = 0$ A1

$(2\cos x - 1)(\cos x + 1) = 0$ (A1)

$\cos x = \dfrac{1}{2}$: $x = \dfrac{\pi}{3},\ \dfrac{5\pi}{3}$ A1A1

$\cos x = -1$: $x = \pi$ A1

Question 5 — 5 marks

Answer: $x = \dfrac{\pi}{4},\ \dfrac{3\pi}{4}$

$2\sin x \cos x = \dfrac{\sin x}{\cos x}$ M1

$\sin x(2\cos^2 x - 1) = 0$ A1

$\sin x \neq 0$ for $0 < x < \pi$, so $\cos 2x = 0$ or $\cos x = \pm\dfrac{1}{\sqrt{2}}$ R1

$x = \dfrac{\pi}{4},\ \dfrac{3\pi}{4}$ A1A1

Question 6 — 6 marks

Answer: $x = 0,\ \dfrac{2\pi}{3},\ \dfrac{4\pi}{3},\ 2\pi$

$2\cos^2 x - 1 = \cos x$ M1

$2\cos^2 x - \cos x - 1 = 0$ A1

$(2\cos x + 1)(\cos x - 1) = 0$ (A1)

$\cos x = 1$: $x = 0,\ 2\pi$ A1

$\cos x = -\dfrac{1}{2}$: $x = \dfrac{2\pi}{3},\ \dfrac{4\pi}{3}$ A1A1

Question 7 — 5 marks

Answer: (a) $\frac{7}{9}$; (b) $\frac{4\sqrt{2}}{9}$

(a)

$1 - 2\sin^2\theta = 1 - \dfrac{2}{9}$ M1

$\dfrac{7}{9}$ A1

(b)

$\cos\theta = \dfrac{2\sqrt{2}}{3}$ A1

$2 \times \dfrac{1}{3} \times \dfrac{2\sqrt{2}}{3}$ M1

$\dfrac{4\sqrt{2}}{9}$ A1

Question 8 — 5 marks

Answer: (b) $-\frac{\sqrt{3}}{2}$

(a)

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$ M1

$= \sin^2 x - \cos^2 x$ A1

$= -(\cos^2 x - \sin^2 x) = -\cos 2x$ A1 AG

(b)

$-\cos\dfrac{\pi}{6}$ (M1)

$-\frac{\sqrt{3}}{2}$ **A1**

Question 9

6 marks

Answer: (a) $-7/8$; (b) $\sqrt{15}/8$

1. Use $\cos(2x) = 2\cos^2 x - 1$ **M1**
   $\cos(2x) = 2\left(\frac{1}{16}\right) - 1 = -\frac{7}{8}$ **A1**
2. Use Pythagoras: $\sin^2 x = 1 - \frac{1}{16} = \frac{15}{16}$
   M1
   x is acute, so take the positive root:
   $\sin x = \frac{\sqrt{15}}{4}$ **A1**
   Use
   $\sin(2x) = 2\sin x \cos x = 2\left(\frac{\sqrt{15}}{4}\right)\left(\frac{1}{4}\right)$
   M1
   $\sin(2x) = \frac{\sqrt{15}}{8}$ **A1**

Question 10

6 marks

Answer: (b) $x = \frac{7\pi}{6},\ \frac{11\pi}{6}$

(a)

$1 - 2\sin^2 x + 3\sin x + 1 = 0$ **M1**

$-2\sin^2 x + 3\sin x + 2 = 0$, so

$2\sin^2 x - 3\sin x - 2 = 0$ **A1 AG**

(b)

$(2\sin x + 1)(\sin x - 2) = 0$ **M1**

$\sin x = 2$ is impossible, so $\sin x = -\frac{1}{2}$ **R1**

$x = \frac{7\pi}{6},\ \frac{11\pi}{6}$ **A1A1**

Every solution is also on ibmathrevision.com/slaa/trigonometric-identities-exam-practice
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