IB MATHS AA SL · UNIT 3

Trigonometric Functions: exam practice

10 questions · 50 marks · worked solutions and markschemes

Name _____ Date _____ Score / 50

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/trigonometric-functions-exam-practice

1. **4 marks · Paper 1 style · no calculator**

The graph of $y = a\sin(bx) + c$, where $a, b > 0$, has a maximum value of 7, a minimum value of -1 and period 4π.

Find the values of a, b and c. **[4]**

2. **4 marks · Paper 1 style · no calculator**

Let $f(x) = 3\cos(2x) + 1$.

(a) Write down the amplitude of f. **[1]**

(b) Find the period of f. **[1]**

(c) Write down the range of f. **[2]**

3. **5 marks · Paper 2 style · calculator**

The height, h in metres, of a passenger on a Ferris wheel above the ground is modelled by the function:

$$h(t) = 20\sin\left(\frac{\pi}{30}t - \frac{\pi}{2}\right) + 25$$

where t is the time in seconds after the ride begins.

1. Find the maximum height and the minimum height of the passenger above the ground.
2. Calculate the time it takes for the Ferris wheel to complete exactly one full revolution.

4. **4 marks · Paper 1 style · no calculator**

Let $g(x) = 2\sin\left(x - \frac{\pi}{4}\right)$, for $0 \leq x \leq 2\pi$.

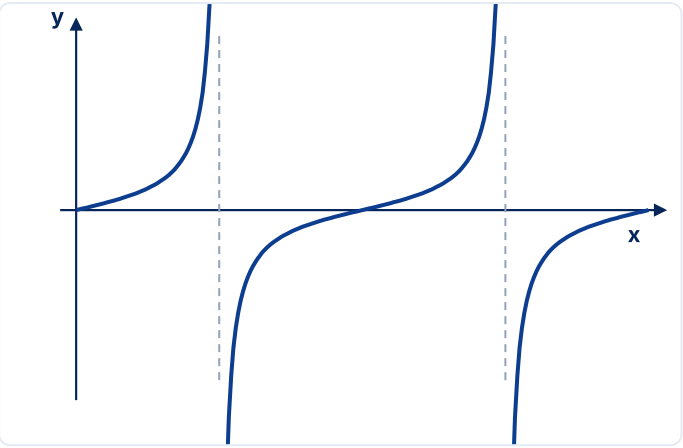
(a) Find the x-intercepts of the graph of g. **[2]**

(b) Write down the coordinates of the maximum point. **[2]**

5. **5 marks · Paper 1 style · no calculator**

The graph of $y = \tan\left(\frac{x}{2}\right)$ is shown below for $0 \leq x \leq 4\pi$.

1. State the period of the function.
2. Write down the exact equations of the two vertical asymptotes shown in the domain $0 \leq x \leq 4\pi$.



6. **5 marks · Paper 2 style · calculator**

The depth of water in a harbour, d metres, t hours after midnight is modelled by

$d(t) = 1.8\cos\left(\frac{\pi t}{6}\right) + 4.2$, for $0 \leq t \leq 12$.

(a) Write down the maximum depth. **[1]**

(b) Find the times when the depth is 5 m. **[3]**

(c) Find the length of time for which the depth is less than 5 m. **[1]**

7. **5 marks · Paper 1 style · no calculator**

Let $f(x) = 2\sin(3x)$, for $0 \leq x \leq \pi$.

(a) Write down the period of f. **[1]**

(b) Solve $f(x) = 1$. **[4]**

8. **6 marks · Paper 2 style · calculator**

A Ferris wheel with a diameter of $80\ \text{m}$ rotates at a constant speed. The bottom of the wheel is $4\ \text{m}$ above the ground. The wheel completes one full revolution in 20 minutes. After t minutes, the height of a seat above the ground is given by $h(t) = 44 - a\cos\left(\frac{\pi}{10}t\right)$.

1. Find the value of a.
2. Find the first time t when the seat is exactly $60\ \text{m}$ above the ground.

9. **6 marks · Paper 2 style · calculator**

The number of hours of daylight, S, in a northern city throughout the year is modelled by $S(t) = 3.5\sin\left(\frac{2\pi}{365}(t-80)\right) + 12$, where t is the number of days after January 1st ($1 \leq t \leq 365$).

1. Find the maximum number of hours of daylight, and the day of the year t on which it occurs.
2. A certain crop can only be successfully grown during the part of the year when there are strictly more than 14 hours of continuous daylight. Use your GDC to find the total number of consecutive days this crop can be grown.

10. **6 marks · Paper 2 style · calculator**

The temperature in a town, $T\,°\text{C}$, t hours after midnight is modelled by $T(t) = A\cos\left(B(t-C)\right) + D$, where $A, B > 0$ and $0 \leq C < 24$. The highest temperature is $28\,°\text{C}$ at 14:00 and the lowest is $12\,°\text{C}$ at 02:00. The pattern repeats every 24 hours.

(a) Find the values of A, B, C and D. **[4]**

(b) Find the temperature at 09:00. **[2]**

Answers and worked solutions

Question 1

4 marks

Answer: $a = 4,\ b = \frac{1}{2},\ c = 3$

$a = \frac{7-(-1)}{2} = 4$ **A1**

$c = \frac{7+(-1)}{2} = 3$ **A1**

$\frac{2\pi}{b} = 4\pi$ **(M1)**

$b = \frac{1}{2}$ **A1**

Question 2

4 marks

Answer: (a) 3; (b) π; (c) $-2 \leq f(x) \leq 4$

(a)

3 **A1**

(b)

$\frac{2\pi}{2} = \pi$ **A1**

(c)

$-2 \leq f(x) \leq 4$ **A1A1**

Question 3

5 marks

Answer: (Maximum) $45\ m$; (Minimum) $5\ m$; (b) $60 seconds$

1. The principal axis is $25\ \text{m}$ and the amplitude is $20\ \text{m}$. **(M1)**
   Max height $= d + a = 25 + 20 = 45\ \text{m}$. **A1**
   Min height $= d - a = 25 - 20 = 5\ \text{m}$. **A1**
2. The time for one revolution is the period of the function. **(M1)**
   Period $= \frac{2\pi}{b} = \frac{2\pi}{\pi/30} = 2\pi \times \frac{30}{\pi} = 60$.
   Time $= 60\ \text{seconds}$ (or 1 minute). **A1**

Question 4

4 marks

Answer: (a) $x = \frac{\pi}{4},\ \frac{5\pi}{4}$; (b) $\left(\frac{3\pi}{4},\ 2\right)$

(a)

$x - \frac{\pi}{4} = 0,\ \pi$ **(M1)**

$x = \frac{\pi}{4},\ \frac{5\pi}{4}$ **A1**

(b)

$\left(\frac{3\pi}{4},\ 2\right)$ **A1A1**

Question 5

5 marks

Answer: (a) 2π; (b) $x = \pi, x = 3\pi$

1. For $y = \tan(bx)$ the period is $\frac{\pi}{b}$, here with $b = \frac{1}{2}$: **(M1)**
   Period $= \frac{\pi}{1/2} = 2\pi$. **A1**
2. Vertical asymptotes occur where $\frac{x}{2} = \frac{\pi}{2} + k\pi$, i.e. $x = \pi + 2k\pi$. **(M1)**
   In $0 \leq x \leq 4\pi$: $x = \pi$ **A1**
   and $x = 3\pi$. **A1**

Question 6

5 marks

Answer: (a) $6\ m$; (b) $t = 2.12$ and $t = 9.88$ hours; (c) 7.76 hours

(a)

$6\ m$ **A1**

(b)

$1.8\cos\left(\frac{\pi t}{6}\right) + 4.2 = 5$ **(M1)**

$t = 2.12$ **A1**

$t = 9.88$ **A1**

(c)

$9.879\ldots - 2.120\ldots = 7.76$ hours **A1**

Question 7

5 marks

Answer: (a) $\frac{2\pi}{3}$; (b) $x = \frac{\pi}{18},\ \frac{5\pi}{18},\ \frac{13\pi}{18},\ \frac{17\pi}{18}$

(a)

$\frac{2\pi}{3}$ **A1**

(b)

$\sin 3x = \frac{1}{2}$, with $0 \leq 3x \leq 3\pi$ **M1**

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ **A1**

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ **A2**

Question 8

6 marks

Answer: (a) $a = 40$; (b) $t = 6.31$

1. a is the amplitude, which equals the radius:
   $a = \frac{80}{2}$ **(M1)**
   $a = 40$ **A1**
2. Set up the equation:
   $44 - 40\cos\left(\frac{\pi}{10}t\right) = 60$ **M1**
   $\cos\left(\frac{\pi}{10}t\right) = \frac{44-60}{40} = -0.4$ **A1**
   Inverse cosine (or GDC) for the first positive solution: $\frac{\pi}{10}t = 1.9823\ldots$ **(M1)**
   $t = 6.31$ minutes (3 s.f.) **A1**

Question 9

6 marks

Answer: (Maximum) $15.5 hours$; (Day) $t = 171$; (b) $112 days$

1. Maximum daylight
   $= d + a = 12 + 3.5 = 15.5$ hours. **A1**
   This occurs when the sine function equals 1: **(M1)**
   $\frac{2\pi}{365}(t - 80) = \frac{\pi}{2} \implies t - 80 = \frac{365}{4} = 91.25$ **(M1)**
   $t = 171.25$, so it occurs on day $t = 171$. **A1**
2. Solve $3.5\sin\left(\frac{2\pi}{365}(t - 80)\right) + 12 = 14$ with the GDC (`G-Solv -> ISCT`). **(M1)**
   $t_1 = 115.33\ldots$, $t_2 = 227.16\ldots$, so $S > 14$ for $115.33 < t < 227.17$.
   Whole days in this interval:
   $t = 116, 117, \ldots, 227$, giving
   $227 - 116 + 1 = 112$ days. **A1**
   (Accept an interval length of 111.8 days.)

Question 10

6 marks

Answer: (a) $A = 8,\ B = \frac{\pi}{12},\ C = 14,\ D = 20$; (b) $22.1\,°\text{C}$

(a)

$A = 8$ **A1**

$D = 20$ **A1**

$B = \frac{2\pi}{24} = \frac{\pi}{12}$ **A1**

$C = 14$ **A1**

(b)

$8\cos\left(\frac{\pi}{12}(9 - 14)\right) + 20$ **(M1)**

$22.1\,°\text{C}$ **A1**

Every solution is also on ibmathrevision.com/slaa/trigonometric-functions-exam-practice
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