Transformations of graphs: exam practice

10 questions · 41 marks · worked solutions and markschemes

Name _____ Date _____ Score / 41

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/transformations-of-graphs-exam-practice

1. **3 marks · Paper 1 style · no calculator**

The graph of $y = f(x)$ has a local maximum at the point $P(2, 5)$. Find the coordinates of the local maximum for the graph of $y = 3f(x + 1)$.

2. **2 marks · Paper 1 style · no calculator**

The point $P(3, -4)$ lies on the graph of $y = f(x)$. Write down the coordinates of the image of P on the graph of

a. $y = 2f(x)$; **[1]**

b. $y = f(3x)$. **[1]**

3. **4 marks · Paper 1 style · no calculator**

The point $P(4, -2)$ lies on the curve with equation $y = f(x)$. State the coordinates of the image of point P on the curves with the following equations:

1. $y = f(x) - 3$
2. $y = f(x - 2)$
3. $y = 2f(x)$
4. $y = f(-x)$

4. **4 marks · Paper 1 style · no calculator**

Let $g(x) = \sqrt{x + 4} - 3$, for $x \geq -4$.

(a) Describe the transformation that maps the graph of $y = \sqrt{x}$ onto the graph of g. **[1]**

(b) Write down the range of g. **[1]**

(c) Find the x-intercept of the graph of g. **[2]**

5. **4 marks · Paper 1 style · no calculator**

The graph of $f(x) = x^2$ is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and then stretched vertically by scale factor 2 to give the graph of g.

(a) Find $g(x)$. **[3]**

(b) Write down the coordinates of the vertex of the graph of g. **[1]**

6. **4 marks · Paper 2 style · calculator**

Let $f(x) = x^3 - 2x$. The graph of f is reflected in the x-axis and then translated 2 units up to give the graph of g.

(a) Find $g(x)$. **[2]**

(b) Find the x-intercept of the graph of g. **[2]**

7. **5 marks · Paper 2 style · calculator**

Let $f(x) = \frac{3x}{x-2}$ for $x \neq 2$.

1. State the equations of the vertical and horizontal asymptotes of the graph of $y = f(x)$.
2. The function g is defined as $g(x) = f(x+4) - 1$. Determine the equations of the asymptotes of $g(x)$.
3. Use your GDC to find the coordinates where the graph of $g(x)$ intersects the x-axis.

8. **5 marks · Paper 1 style · no calculator**

The graph of $y = f(x)$ has a minimum point at $(2, -5)$.

(a) Write down the coordinates of the minimum point on the graph of $y = f(x-1) + 4$. **[2]**

(b) Write down the coordinates of the maximum point on the graph of $y = -2f(x)$. **[2]**

(c) Write down the coordinates of the minimum point on the graph of $y = f(-x)$. **[1]**

9. **6 marks · Paper 1 style · no calculator**

The point $A(6, 4)$ lies on the graph of $y = f(x)$. The graph of $y = g(x)$ is given by $g(x) = -f(2x) + 1$.

(a) Find the coordinates of the image of A on the graph of g. **[3]**

(b) Describe fully the three transformations that map the graph of f onto the graph of g. **[3]**

10. **4 marks · Paper 1 style · no calculator**

Let $f(x) = \ln x$ and $g(x) = \ln(3x)$, for $x > 0$.

(a) Describe the horizontal stretch that maps the graph of f onto the graph of g. **[1]**

(b) Show that the same graph is obtained by a vertical translation, and state the translation vector. **[3]**

Answers and worked solutions

Question 1

3 marks

Answer: $(1, 15)$

Translation 1 unit left: x—coordinate $2 - 1 = 1$. **M1**

Vertical stretch with scale factor 3: y—coordinate $5 \times 3 = 15$. **M1**

Local maximum at $(1, 15)$. **A1**

Question 2

2 marks

Answer: (a) $(3, -8)$; (b) $(1, -4)$

a. $(3, -8)$ **A1**

b. x—coordinate divided by 3: $(1, -4)$ **A1**

Question 3

4 marks

Answer: (a) $(4, -5)$; (b) $(6, -2)$; (c) $(4, -4)$; (d) $(-4, -2)$

1. $f(x) - 3$ represents a vertical translation 3 units down. $(4, -2 - 3) \implies (4, -5)$. **A1**
2. $f(x - 2)$ represents a horizontal translation 2 units right. $(4 + 2, -2) \implies (6, -2)$. **A1**
3. $2f(x)$ represents a vertical stretch by scale factor 2. $(4, 2 \times -2) \implies (4, -4)$. **A1**
4. $f(-x)$ represents a horizontal reflection in the y—axis. $(-4, -2)$. **A1**

Question 4

4 marks

Answer: (b) $g(x) \geq -3$; (c) 5

(a)

translation by $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$ **A1**

(b)

$g(x) \geq -3$ **A1**

(c)

$\sqrt{x + 4} = 3$ **(M1)**

$x = 5$ **A1**

Question 5

4 marks

Answer: (a) $g(x) = 2(x - 3)^2 - 4$; (b) $(3, -4)$

(a)

translation: $(x - 3)^2 - 2$ **(A1)**

stretch: $2\left((x - 3)^2 - 2\right)$ **(M1)**

$g(x) = 2(x - 3)^2 - 4$ **A1**

(b)

$(3, -4)$ **A1**

Question 6

4 marks

Answer: (a) $-x^3 + 2x + 2$; (b) $x = 1.77$

(a)

$-f(x) + 2$ **(M1)**

$g(x) = -x^3 + 2x + 2$ **A1**

(b)

$-x^3 + 2x + 2 = 0$ **(M1)**

$x = 1.77$ **A1**

Question 7

5 marks

Answer: (a) $x = 2, y = 3$; (b) $x = -2, y = 2$; (c) $(-5, 0)$

1. Vertical asymptote (denominator zero): $x = 2$. **A1**
   Horizontal asymptote (ratio of leading coefficients): $y = 3$. **A1**
2. $f(x + 4)$ translates the graph 4 units left and -1 translates it 1 unit down.
   Vertical asymptote: $x = 2 - 4 = -2$. **A1**
   Horizontal asymptote: $y = 3 - 1 = 2$. **A1**
3. **CG50:** `MENU 5. Y1 = 3(x+4)/(x+2) - 1. G-Solv -> ROOT` gives $x = -5$.
   Coordinates: $(-5, 0)$. **A1**
   (Check: $3(x + 4) = x + 2 \implies x = -5$.)

Question 8

5 marks

Answer: (a) $(3, -1)$; (b) $(2, 10)$; (c) $(-2, -5)$

(a)

$(3, -1)$ **A1A1**

(b)

$(2, 10)$ **A1A1**

(c)

$(-2, -5)$ **A1**

Question 9

6 marks

Answer: (a) $(3, -3)$

(a)

$x-$coordinate $\frac{6}{2} = 3$ **A1**

$y-$coordinate $-4 + 1$ **(M1)**

$(3, -3)$ **A1**

(b)

horizontal stretch, scale factor $\frac{1}{2}$ **A1**

reflection in the $x-$axis **A1**

translation by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (after the reflection) **A1**

Question 10

4 marks

Answer: (b) translation $\begin{pmatrix} 0 \\ \ln 3 \end{pmatrix}$

(a)

horizontal stretch, scale factor $\frac{1}{3}$ **A1**

(b)

$\ln(3x) = \ln 3 + \ln x$ **M1**

so $g(x) = f(x) + \ln 3$ **A1**

translation $\begin{pmatrix} 0 \\ \ln 3 \end{pmatrix}$ **A1**

Every solution is also on ibmathrevision.com/slaa/transformations-of-graphs-exam-practice

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