IB MATHS AA SL · UNIT 3

The unit circle and exact values of trigonometric ratios: exam practice

10 questions · 47 marks · worked solutions and markschemes

Name _____ Date _____ Score / 47

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/the-unit-circle-and-exact-values-of-trigonometric-ratios-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Show that $\sin\frac{\pi}{3}\cos\frac{\pi}{6}+\cos\frac{\pi}{3}\sin\frac{\pi}{6}=1$. **[3]**

2. **5 marks · Paper 1 style · no calculator**

Solve the equation $3\tan^2 x-1=0$ for $0°\leq x\leq 360°$.

3. 5 marks · Paper 1 style · no calculator

Given that $\tan\theta = -2$ and $\frac{\pi}{2} < \theta < \pi$,

find the exact values of $\cos\theta$ and $\sin\theta$. **[5]**

4. 3 marks · Paper 1 style · no calculator

Write down the exact value of

(a) $\sin\frac{5\pi}{6}$ **[1]**

(b) $\cos\frac{7\pi}{4}$ **[1]**

(c) $\tan\frac{2\pi}{3}$ **[1]**

5. 4 marks · Paper 1 style · no calculator

Given that $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the exact values of $\cos\theta$ and $\tan\theta$.

6. **5 marks · Paper 1 style · no calculator**

Using your knowledge of the unit circle, write down the exact values of the following trigonometric ratios:

1. $\sin\left(\frac{\pi}{6}\right)$
2. $\cos\left(\frac{3\pi}{4}\right)$
3. $\tan\left(\frac{4\pi}{3}\right)$
4. $\cos\left(\frac{3\pi}{2}\right)$
5. $\sin\left(\frac{11\pi}{6}\right)$

7. **4 marks · Paper 1 style · no calculator**

Consider an acute angle θ measured in radians such that $\sin\theta = p$. Find expressions for the following in terms of p:

1. $\sin(\pi - \theta)$
2. $\cos(\pi + \theta)$
3. $\tan(2\pi - \theta)$

8. **4 marks · Paper 1 style · no calculator**

Given that $\cos\theta = -\frac{5}{13}$ and $\pi < \theta < \frac{3\pi}{2}$,

(a) find the exact value of $\sin\theta$; **[3]**

(b) find the exact value of $\tan\theta$. **[1]**

9. **6 marks · Paper 1 style · no calculator**

The point $P(-0.6,\ 0.8)$ lies on the unit circle, and θ is the angle measured anticlockwise from the positive x-axis to $[OP]$.

(a) Write down the value of $\cos\theta$ and of $\tan\theta$. **[2]**

(b) Find the value of $\sin(\theta+\pi)$. **[2]**

(c) Find the value of $\cos(\pi-\theta)$. **[2]**

10. **8 marks · Paper 1 style · no calculator**

The angle θ satisfies $\frac{\pi}{2}<\theta<\pi$ and $\sin\theta=\frac{5}{13}$.

a. Find the exact value of $\cos\theta$. **[3]**

b. Find the exact value of $\tan\theta$. **[1]**

c. Find the exact value of $\sin(\pi+\theta)$ and of $\cos(2\pi-\theta)$. **[2]**

d. Find the exact value of $\sin 2\theta$. **[2]**

Answers and worked solutions

Question 1

3 marks

Answer: a proof: the worked solution above is the full answer

$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$ A1A1

$= \frac{3}{4} + \frac{1}{4} = 1$ A1 AG

Question 2

5 marks

Answer:

$x = 30°, x = 150°, x = 210°, x = 330°$

Rearrange:

$3\tan^2 x - 1 = 0 \implies 3\tan^2 x = 1 \implies \tan^2 x = \frac{1}{3}$

.

Take the square root of both sides: $\tan x = \pm\frac{1}{\sqrt{3}}$.

(M1)A1

The reference angle is $30°$. Since the ratio is $\pm$, there is a solution in all four quadrants. M1

$Q1$: $x = 30°$; $Q3$: $x = 180° + 30° = 210°$. A1

$Q2$: $x = 180° - 30° = 150°$; $Q4$: $x = 360° - 30° = 330°$. A1

Question 3

5 marks

Answer: $\cos\theta = -\frac{1}{\sqrt{5}}$, $\sin\theta = \frac{2}{\sqrt{5}}$

right-angled triangle with sides 1, 2 and hypotenuse $\sqrt{5}$, or $1 + \tan^2\theta = \frac{1}{\cos^2\theta}$ M1

$\cos^2\theta = \frac{1}{5}$ A1

second quadrant: $\cos\theta < 0$, $\sin\theta > 0$ R1

$\cos\theta = -\frac{1}{\sqrt{5}}$ A1

$\sin\theta = \frac{2}{\sqrt{5}}$ A1

Question 4

3 marks

Answer: (a) $\frac{1}{2}$; (b) $\frac{\sqrt{2}}{2}$; (c) $-\sqrt{3}$

(a)

$\frac{1}{2}$ A1

(b)

$\frac{\sqrt{2}}{2}$ A1

(c)

$-\sqrt{3}$ A1

Question 5

4 marks

Answer: $(\cos\theta)\ -4/5$; $(\tan\theta)\ -3/4$

Since $\frac{\pi}{2} < \theta < \pi$, the angle θ is in the second quadrant. (R1)

In Quadrant 2, sine is positive, but cosine and tangent are negative.

Using the Pythagorean identity:

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$

. (M1)

Therefore, $\cos\theta = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$. A1

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$: $\tan\theta = \frac{3/5}{-4/5} = -\frac{3}{4}$. A1

Question 6

5 marks

Answer: (a) $1/2$; (b) $-\sqrt{2}/2$; (c) $\sqrt{3}$; (d) 0; (e) $-1/2$

Recall standard unit circle coordinates.

1. $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$. A1
2. $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ (or $-\frac{1}{\sqrt{2}}$). A1
3. $\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$. A1
4. $\cos\left(\frac{3\pi}{2}\right) = 0$. A1
5. $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$. A1

Question 7

4 marks

Answer: (a) p; (b) $-\sqrt{1-p^2}$; (c) $-p/\sqrt{1-p^2}$

Since θ is acute ($0 < \theta < \frac{\pi}{2}$) and $\sin\theta = p$, we have $p > 0$.

From the Pythagorean identity, $\cos\theta = \sqrt{1-p^2}$. (M1)

1. $\sin(\pi-\theta)=\sin\theta=p$. **A1**
2. $\cos(\pi+\theta)=-\cos\theta=-\sqrt{1-p^2}$. **A1**
3. $\tan(2\pi-\theta)=-\tan\theta=-\frac{\sin\theta}{\cos\theta}=-\frac{p}{\sqrt{1-p^2}}$. **A1**

Question 8

4 marks

Answer: (a) $-\frac{12}{13}$; (b) $\frac{12}{5}$

(a)

$\sin^2\theta=1-\frac{25}{169}=\frac{144}{169}$ **M1**

third quadrant, so $\sin\theta<0$ **R1**

$\sin\theta=-\frac{12}{13}$ **A1**

(b)

$\frac{12}{5}$ **A1**

Question 9

6 marks

Answer: (a) $\cos\theta=-0.6$, $\tan\theta=-\frac{4}{3}$; (b) -0.8; (c) 0.6

(a)

$\cos\theta=-0.6$ **A1**

$\tan\theta=\frac{0.8}{-0.6}=-\frac{4}{3}$ **A1**

(b)

$\sin(\theta+\pi)=-\sin\theta$ **(M1)**

-0.8 **A1**

(c)

$\cos(\pi-\theta)=-\cos\theta$ **(M1)**

0.6 **A1**

Question 10

8 marks

Answer: (a) $-12/13$; (b) $-5/12$; (c) $\sin(\pi+\theta)$ $-5/13$; (c) $\cos(2\pi-\theta)$ $-12/13$; (d) $-120/169$

a. $\cos^2\theta=1-\frac{25}{169}$ **(M1)**

$\cos\theta=-\frac{12}{13}$ (second quadrant) **A1R1**

b. $-\frac{5}{12}$ **A1**

c. $\sin(\pi+\theta)=-\frac{5}{13}$ **A1**

$\cos(2\pi-\theta)=\cos\theta=-\frac{12}{13}$ **A1**

d. $2\times\frac{5}{13}\times\left(-\frac{12}{13}\right)$ **(M1)**

$=-\frac{120}{169}$ **A1**

Every solution is also on ibmathrevision.com/slaa/the-unit-circle-and-exact-values-of-trigonometric-ratios-exam-practice
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