IB MATHS AA SL · UNIT 3

Solving Trigonometric Equations: exam practice

10 questions · 47 marks · worked solutions and markschemes

Name _____ Date _____ Score / 47

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/solving-trigonometric-equations-exam-practice

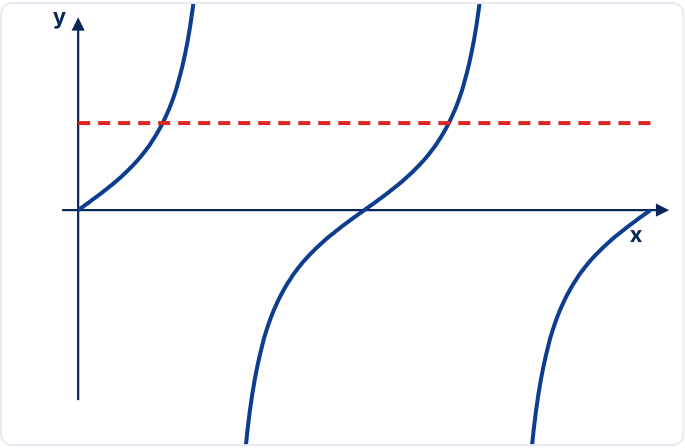
1. **4 marks · Paper 1 style · no calculator**

Solve the equation $2\sin x - \sqrt{3} = 0$ for the interval $0 \leq x \leq 2\pi$. Give your answers exactly in radians.

2. **4 marks · Paper 2 style · calculator**

The graphs of $y = 3\tan x$ and $y = 4$ are shown below for $0 \leq x \leq 2\pi$.

Use your graphic display calculator to solve the equation $3\tan x = 4$ for $0 \leq x \leq 2\pi$. Give your answers correct to three significant figures.



3. **4 marks · Paper 1 style · no calculator**

Solve the trigonometric equation $2\sin^2 x + \sin x - 1 = 0$ for $0 \leq x \leq 2\pi$. Give your exact answers in radians.

4. **4 marks · Paper 1 style · no calculator**

Solve the equation $\tan^2 x - 3 = 0$ for $0 \leq x \leq 2\pi$. Give your answers exactly in radians.

5. **4 marks · Paper 1 style · no calculator**

Solve $2\cos(2x) + 1 = 0$ for $0 \leq x \leq \pi$. **[4]**

6. **4 marks · Paper 2 style · calculator**

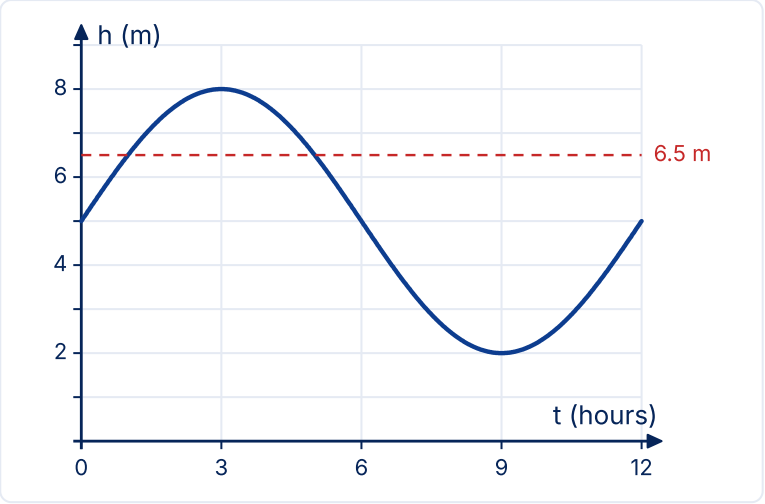
Solve $\tan(x - 0.5) = 2$ for $0 \leq x \leq 2\pi$. **[4]**

7. **6 marks · Paper 1 style · no calculator**

Solve the equation $2\cos^2(x) - \sqrt{3}\cos(x) = 0$ for $0 \leq x \leq 2\pi$. Give your answers exactly in radians.

8. **5 marks · Paper 2 style · calculator**

The height of the tide in a harbour, h in metres, at time t hours after midnight is modelled by $h(t) = 3\sin\left(\frac{\pi}{6}t\right) + 5$.



A large cargo ship can only safely navigate the harbour when the tide height is strictly greater than $6.5\ \text{m}$ (shown by the dashed line). Use your GDC to solve $h(t) = 6.5$ and find the total amount of time during the first 12 hours ($0 \leq t \leq 12$) when the ship can safely navigate the harbour.

9. **6 marks · Paper 1 style · no calculator**

Solve $2\cos^2 x = 3\sin x$ for $0 \leq x \leq 2\pi$. **[6]**

10. 6 marks · Paper 2 style · calculator

Solve $3\sin 2x = 2\cos x$ for $0 \leq x \leq 2\pi$. **[6]**

Answers and worked solutions

Question 1 — 4 marks

Answer: $x = \pi/3, x = 2\pi/3$

$2\sin x - \sqrt{3} = 0 \implies 2\sin x = \sqrt{3} \implies \sin x = \frac{\sqrt{3}}{2}$. **A1**

The principal reference angle is $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$. **(M1)**

Sine is positive in Quadrants 1 and 2.

$Q1$: $x = \frac{\pi}{3}$. **A1**

$Q2$: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. **A1**

Question 2 — 4 marks

Answer: $x = 0.927, x = 4.07$

CG50: Ensure calculator is in **Radian** mode. `MENU 5 (Graph)`.

Plot `Y1 = 3tan(x)` and `Y2 = 4`. `DRAW`. **(M1)**

Set V-Window from $x = 0$ to $x = 2\pi$.

Press `G-Solv (F5) -> ISCT (F5)`.

First intersection:

$x = 0.92729\ldots \implies x = 0.927$ radians. **A1**

Since $\tan x$ has period π, the second solution is $0.92729\ldots + \pi$. **(M1)**

Second intersection:

$x = 4.0688\ldots \implies x = 4.07$ radians. **A1**

Question 3 — 4 marks

Answer: $x = \pi/6, x = 5\pi/6, x = 3\pi/2$

Factorise the quadratic in $\sin x$:

$(2\sin x - 1)(\sin x + 1) = 0$ **M1**

So $\sin x = \frac{1}{2}$ or $\sin x = -1$ **A1**

Solve $\sin x = \frac{1}{2}$: $x = \frac{\pi}{6}, \frac{5\pi}{6}$ **A1**

Solve $\sin x = -1$: $x = \frac{3\pi}{2}$ **A1**

Question 4 — 4 marks

Answer:

$x = \pi/3, x = 2\pi/3, x = 4\pi/3, x = 5\pi/3$

Rearrange: $\tan^2 x - 3 = 0 \implies \tan^2 x = 3$.

Take square roots (both signs): $\tan x = \pm\sqrt{3}$. **(M1)A1**

The reference angle is $\frac{\pi}{3}$. Because it is $\pm$, there is a solution in all four quadrants.

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$. **A2**

(Award only one of these two marks if just the positive-root solutions $\frac{\pi}{3}, \frac{4\pi}{3}$ are found).

Question 5 — 4 marks

Answer: $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

$\cos 2x = -\dfrac{1}{2}$ **(A1)**

$2x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ **(M1)(A1)**

$x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ **A1**

Question 6 — 4 marks

Answer: $x = 1.61$ or $x = 4.75$

$x - 0.5 = \arctan 2 = 1.107\ldots$ **(M1)(A1)**

$x = 1.61$ **A1**

$x = 1.61 + \pi = 4.75$ **A1**

Question 7 — 6 marks

Answer:

$x = \pi/2, x = 3\pi/2, x = \pi/6, x = 11\pi/6$

Factorise: $\cos x(2\cos x - \sqrt{3}) = 0$ **M1**

$\cos x = 0$ or $\cos x = \frac{\sqrt{3}}{2}$ **A1**

$\cos x = 0 \implies x = \frac{\pi}{2}$ **A1**

and $x = \frac{3\pi}{2}$ **A1**

$\cos x = \frac{\sqrt{3}}{2} \implies x = \frac{\pi}{6}$ **A1**

or $x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ **A1**

Question 8 — 5 marks

Answer: $4 hours$

We need to solve $3\sin\left(\frac{\pi}{6}t\right) + 5 = 6.5$. **(M1)**

CG50: `MENU 5`. Plot `Y1 = 3sin(\pi/6 x) + 5` and `Y2 = 6.5`. `DRAW`. **(M1)**

Press `G-Solv -> ISCT`.

First intersection: $t_1 = 1$ hour. **A1**

Second intersection: $t_2 = 5$ hours. **A1**

The tide is strictly above 6.5 m between $t = 1$ and $t = 5$.

Total time $= 5 - 1 = 4$ hours. **A1**

Question 9

6 marks

Answer: $x = \frac{\pi}{6},\ \frac{5\pi}{6}$

$2(1 - \sin^2 x) = 3\sin x$ **M1**

$2\sin^2 x + 3\sin x - 2 = 0$ **A1**

$(2\sin x - 1)(\sin x + 2) = 0$ **(A1)**

$\sin x = -2$ has no solution **R1**

$x = \frac{\pi}{6},\ \frac{5\pi}{6}$ **A1A1**

Question 10

6 marks

Answer: $x = \frac{\pi}{2},\ \frac{3\pi}{2},\ 0.340,\ 2.80$

$6\sin x \cos x = 2\cos x$ **M1**

$2\cos x(3\sin x - 1) = 0$ **A1**

$\cos x = 0$: $x = \frac{\pi}{2},\ \frac{3\pi}{2}$ **A1A1**

$\sin x = \frac{1}{3}$: $x = 0.340,\ 2.80$ **A1A1**

Every solution is also on ibmathrevision.com/slaa/solving-trigonometric-equations-exam-practice
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