IB MATHS AA SL · UNIT 1

Sequences and Series: exam practice

10 questions · 49 marks · worked solutions and markschemes

Name _____ Date _____ Score / 49

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/sequences-and-series-exam-practice

1. **3 marks · Paper 1 style · no calculator**

The sum of the first n terms of a sequence is given by the formula $S_n = 3n^2 - 2n$. Find the exact value of the 10th term in the sequence, u_{10}.

2. **3 marks · Paper 1 style · no calculator**

The sum of the first 16 terms of an arithmetic sequence is 920. Find the common difference, d, of the sequence if the first term is 27.5.

3. **3 marks · Paper 1 style · no calculator**

Find the value of $\sum_{r=1}^{20}(3r+2)$. **[3]**

4. **4 marks · Paper 1 style · no calculator**

Evaluate exactly the sum given by $\sum_{k=1}^{15}(4k-5)$.

5. **4 marks · Paper 2 style · calculator**

An infinite geometric series has a sum to infinity of 400. The common ratio of the series is 0.8.

1. Find the first term of the series, u_1.
2. Find the sum of the first 8 terms.

6. **6 marks · Paper 2 style · calculator**

The second term, u_2, of a geometric sequence is 44 and the third term, u_3, is 55.

1. Find the common ratio, r, of the sequence.
2. Find the first term of the sequence, u_1.
3. Find S_5, the sum of the first 5 terms of the sequence.

7. **6 marks · Paper 1 style · no calculator**

In a geometric sequence, the third term is 160 and the common ratio is $\frac{1}{4}$.

1. Find the first term, u_1.
2. Find u_6.
3. Find the value of the infinite sum of the sequence.

8. **6 marks · Paper 1 style · no calculator**

The fourth term of an arithmetic sequence is 17 and the tenth term is 47.

(a) Find the common difference and the first term. **[4]**

(b) Find the sum of the first 30 terms. **[2]**

9. **6 marks · Paper 2 style · calculator**

A ball is dropped from a height of 8 m. After each bounce it rises to 70% of the height it fell from.

(a) Find the height the ball reaches after the third bounce. **[2]**

(b) Find the total distance, up and down, that the ball travels before it comes to rest. **[4]**

10. **8 marks · Paper 1 style · no calculator**

The first three terms of a geometric sequence are $k-3$, $k+1$ and $3k-1$, where $k>3$.

(a) Show that $k^2-6k+1=0$. **[3]**

(b) Find the exact value of k. **[2]**

(c) Find the common ratio, and explain why the series has no sum to infinity. **[3]**

Answers and worked solutions

Question 1

3 marks

Answer: 55

Use $u_{10} = S_{10} - S_9$ **M1**
$S_{10} = 3(10)^2 - 2(10) = 280$ and
$S_9 = 3(9)^2 - 2(9) = 225$ **(A1)**
$u_{10} = 280 - 225 = 55$ **A1**

Question 2

3 marks

Answer: $d = 4$

$S_{16} = \frac{16}{2}(2(27.5) + 15d) = 920$. **(M1)**
$8(55 + 15d) = 920 \implies 55 + 15d = 115$. **(A1)**
$15d = 60 \implies d = 4$. **A1**

Question 3

3 marks

Answer: 670

arithmetic: first term 5, last term 62, 20 terms **(M1)** **(A1)**

$$\frac{20}{2}(5 + 62) = 670$$ **A1**

Question 4

4 marks

Answer: 405

Recognise an arithmetic series ($d = 4$) with first term $u_1 = 4(1) - 5 = -1$ **M1**
Last term $u_{15} = 4(15) - 5 = 55$ **A1**
Substitute into $S_n = \frac{n}{2}(u_1 + u_n)$:
$S_{15} = \frac{15}{2}(-1 + 55)$ **M1**
$= 15 \times 27 = 405$ **A1**

Question 5

4 marks

Answer: (a) $u_1 = 80$; (b) 333

1. Substitute into $S_\infty = \frac{u_1}{1-r}$: $400 = \frac{u_1}{1-0.8}$ **M1**
   $u_1 = 400 \times 0.2 = 80$ **A1**
2. Use the finite sum formula: $S_8 = \frac{80(1-0.8^8)}{1-0.8}$ **M1**
   $S_8 = 332.89\ldots \approx 333$ **A1**

Question 6

6 marks

Answer: (a) $r = 1.25$; (b) $u_1 = 35.2$; (c) 289

1. Divide consecutive terms: $r = \frac{u_3}{u_2} = \frac{55}{44}$. **M1**
   $r = \frac{5}{4} = 1.25$. **A1**
2. $u_1 = \frac{u_2}{r} = \frac{44}{1.25}$. **(M1)**
   $u_1 = 35.2$. **A1**
3. Use the geometric sum formula:
   $S_5 = \frac{35.2(1.25^5 - 1)}{1.25 - 1}$. **(M1)**
   $S_5 = 288.8875$ (289 to 3 s.f.). **A1**

Question 7

6 marks

Answer: (a) 2560; (b) 2.5; (c) $10240/3$

1. $u_3 = u_1 r^2 \implies 160 = u_1\left(\frac{1}{4}\right)^2$. **(M1)**
   $160 = u_1\left(\frac{1}{16}\right) \implies u_1 = 2560$. **A1**
2. $u_6 = 2560\left(\frac{1}{4}\right)^5$. **(M1)**
   $u_6 = 2.5$. **A1**
3. $S_\infty = \frac{2560}{1-0.25} = \frac{2560}{0.75}$. **(M1)**
   $S_\infty = \frac{10240}{3} \approx 3413.33$. **A1**

Question 8

6 marks

Answer: (a) $d = 5,\ u_1 = 2$; (b) 2235

(a)

$u_1 + 3d = 17$, $u_1 + 9d = 47$ **(M1)**
$6d = 30$ **(A1)**
$d = 5$ **A1**
$u_1 = 2$ **A1**

(b)

$$S_{30} = \frac{30}{2}(2 \times 2 + 29 \times 5)$$ **(M1)**
$= 2235$ **A1**

Question 9

6 marks

Answer: (a) $2.744\ m$; (b) $45.3\ m$

(a)

8×0.7^3 **(M1)**

$2.744\,m$ **A1**

(b)

downward: $8 + 8(0.7) + 8(0.7)^2 + \ldots$, upward: $8(0.7) + 8(0.7)^2 + \ldots$ **(M1)**

$S_\infty = \dfrac{8(0.7)}{1 - 0.7} = \dfrac{56}{3}$ for the rises **(A1)**

total $= 8 + 2 \times \dfrac{56}{3}$ **(M1)**

$= \dfrac{136}{3} = 45.3\,m$ **A1**

Question 10

8 marks

Answer: (b) $k = 3 + 2\sqrt{2}$; (c) $r = 1 + \sqrt{2}$

(a)

$\dfrac{k+1}{k-3} = \dfrac{3k-1}{k+1}$ **M1**

$(k+1)^2 = (3k-1)(k-3)$, so

$k^2 + 2k + 1 = 3k^2 - 10k + 3$ **A1**

$2k^2 - 12k + 2 = 0$, so $k^2 - 6k + 1 = 0$ **A1 AG**

(b)

$k = \dfrac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$ **(M1)**

$k > 3$, so $k = 3 + 2\sqrt{2}$ **A1**

(c)

$r = \dfrac{k+1}{k-3} = \dfrac{4 + 2\sqrt{2}}{2\sqrt{2}}$ **M1**

$= 1 + \sqrt{2}$ **A1**

$|r| > 1$, so the series diverges **R1**

Every solution is also on ibmathrevision.com/slaa/sequences-and-series-exam-practice

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