IB MATHS AA SL · UNIT 5

Rules of Differentiation: exam practice

12 questions · 52 marks · worked solutions and markschemes

Name _____ Date _____ Score / 52

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/rules-of-differentiation-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Let $f(x) = \left(x^2 + 1\right)^3$. Find $f'(1)$.

2. **3 marks · Paper 1 style · no calculator**

Let $y = \dfrac{3x - 1}{x + 2}$. Show that $\dfrac{dy}{dx} = \dfrac{7}{(x + 2)^2}$. **[3]**

3. **4 marks · Paper 1 style · no calculator**

Find the exact equation of the tangent line to the curve $f(x) = \ln(2x + 1)$ at the point where $x = 0$.

4. **4 marks · Paper 1 style · no calculator**

Find the exact coordinates of the stationary point on the curve $f(x) = \frac{e^x}{x+1}$, where $x \neq -1$.

5. **4 marks · Paper 1 style · no calculator**

The equation of a curve is given by $y = \frac{3}{2}x^2 - 15x + 2$.

1. Find an expression for $\frac{dy}{dx}$.
2. Find the exact coordinates of the point on the curve where the gradient of the tangent is equal to 0.

6. **4 marks · Paper 1 style · no calculator**

Let $f(x) = 3e^x - 4\cos x$.

(a) Find $f'(x)$. **[2]**

(b) Find the gradient of the graph of f at $x = 0$. **[2]**

7. **5 marks · Paper 1 style · no calculator**

The curve C has equation $y = x^2 \ln x$, for $x > 0$.

(a) Use the product rule to find the gradient function of C. **[3]**

(b) Hence find the exact gradient of C at the point where $x = e$. **[2]**

8. **6 marks · Paper 1 style · no calculator**

Let $f(x) = x^3 - 6x^2 + 5$.

(a) Find $f'(x)$. **[2]**

(b) Find the coordinates of the points on the graph of f where the gradient is 15. **[4]**

9. **4 marks · Paper 1 style · no calculator**

Let $f(x) = e^{\sin x}$.

(a) Find $f'(x)$. **[2]**

(b) Find the gradient of the graph of f at $x = \pi$. **[2]**

10. **5 marks · Paper 1 style · no calculator**

Let $f(x) = \sqrt{4x+1}$.

(a) Find $f'(2)$. **[3]**

(b) Find the equation of the tangent to the graph of f at $x = 2$. **[2]**

11. **5 marks · Paper 1 style · no calculator**

Let $f(x) = (2x+1)^3$.

(a) Find $f'(x)$. **[2]**

(b) Find the equation of the normal to the graph of f at $x = 0$. **[3]**

12. **5 marks · Paper 1 style · no calculator**

A curve has the equation $y = e^{-3x} + \ln x$, for $x > 0$.

1. Find $\frac{dy}{dx}$.
2. Find the exact gradient of the normal line to the curve at the point where $x = 1$.

Answers and worked solutions

Question 1

3 marks

Answer: 24

$f'(x) = 3(x^2+1)^2 \times 2x$ (M1)(A1)

$f'(1) = 3 \times 4 \times 2 = 24$ A1

Question 2

3 marks

Answer: a proof: the worked solution above is the full answer

$\frac{3(x+2)-(3x-1)(1)}{(x+2)^2}$ $M1$A1

$= \frac{7}{(x+2)^2}$ A1 AG

Question 3

4 marks

Answer: $y = 2x$

Chain rule: $f'(x) = \frac{2}{2x+1}$ A1

Substitute $x = 0$ to find the gradient: $m = \frac{2}{1} = 2$ M1

Point: $f(0) = \ln 1 = 0$, so $(0, 0)$ A1

Tangent: $y - 0 = 2(x - 0) \implies y = 2x$ A1

Question 4

4 marks

Answer: $(0, 1)$

Quotient rule: $f'(x) = \frac{e^x(x+1)-e^x(1)}{(x+1)^2}$ M1

Simplify: $f'(x) = \frac{xe^x}{(x+1)^2}$ A1

Set $f'(x) = 0$: $xe^x = 0 \implies x = 0$ (since $e^x \neq 0$) A1

$f(0) = \frac{e^0}{1} = 1$, so the stationary point is $(0, 1)$ A1

Question 5

4 marks

Answer: (a) $dy/dx = 3x - 15$; (b) $(5, -35.5)$

1. Using the power rule on polynomials: (M1)
   $\frac{dy}{dx} = 2 \times \left(\frac{3}{2}\right)x^{2-1} - 15$.
   $\frac{dy}{dx} = 3x - 15$. A1
2. Set the gradient to 0:
   $3x - 15 = 0 \implies 3x = 15 \implies x = 5$. (M1)
   Substitute $x = 5$ back into the original equation to find the $y-$coordinate:
   $y = \frac{3}{2}(5)^2 - 15(5) + 2 = \frac{75}{2} - 75 + 2 = 37.5 - 75 + 2 = -35.5$.
   Coordinates: $(5, -35.5)$. A1

Question 6

4 marks

Answer: (a) $f'(x) = 3e^x + 4\sin x$; (b) 3

(a)

$3e^x + 4\sin x$ A1A1

(b)

$3e^0 + 4\sin 0$ (M1)

3 A1

Question 7

5 marks

Answer: (a) $\frac{dy}{dx} = 2x\ln x + x$; (b) $3e$

(a)

product rule M1

$2x\ln x + x^2 \times \frac{1}{x}$ A1

$2x\ln x + x$ A1

(b)

$2e\ln e + e$ (M1)

$3e$ A1

Question 8

6 marks

Answer: (a) $3x^2 - 12x$; (b) $(5, -20)$ and $(-1, -2)$

(a)

$3x^2 - 12x$ A1A1

(b)

$3x^2 - 12x = 15$ M1

$x^2 - 4x - 5 = 0$, $(x-5)(x+1) = 0$ A1

$(5, -20)$ A1

$(-1, -2)$ A1

Question 9

4 marks

Answer: (a) $f'(x) = \cos x\, e^{\sin x}$; (b) -1

(a)

chain rule **(M1)**

$\cos x\, e^{\sin x}$ **A1**

(b)

$\cos \pi \times e^{0}$ **(M1)**

-1 **A1**

Question 10

5 marks

Answer: (a) $\frac{2}{3}$; (b) $y - 3 = \frac{2}{3}(x - 2)$

(a)

$f'(x) = \frac{1}{2}(4x+1)^{-1/2} \times 4$ $M1$**A1**

$f'(2) = \frac{2}{3}$ **A1**

(b)

$f(2) = 3$ **(A1)**

$y - 3 = \frac{2}{3}(x - 2)$ **A1**

Question 11

5 marks

Answer: (a) $6(2x+1)^2$; (b) $y = -\frac{1}{6}x + 1$

(a)

$3(2x+1)^2 \times 2$ **(M1)**

$6(2x+1)^2$ **A1**

(b)

$f'(0) = 6$, $f(0) = 1$ **A1**

gradient of normal $-\frac{1}{6}$ **(M1)**

$y = -\frac{1}{6}x + 1$ **A1**

Question 12

5 marks

Answer: (a) $dy/dx = -3e^{(}-3x) + 1/x$; (b) $-1/(1 - 3e^{(}-3))$

1. Using standard derivatives and the chain rule for the exponential term:
   $\frac{dy}{dx} = -3e^{-3x} + \frac{1}{x}$. $M1$**A1**
2. Gradient of the tangent at $x = 1$:
   $m_t = -3e^{-3(1)} + \frac{1}{1} = 1 - 3e^{-3}$. **(M1)**
   The gradient of the normal is the negative reciprocal: $m_n = -\frac{1}{m_t}$. **(M1)**
   $m_n = \frac{-1}{1-3e^{-3}}$. **A1**

Every solution is also on ibmathrevision.com/slaa/rules-of-differentiation-exam-practice
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