Rational and Reciprocal Functions: exam practice

11 questions · 53 marks · worked solutions and markschemes

Name _____ Date _____ Score / 53

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/rational-and-reciprocal-functions-exam-practice

1. **4 marks · Paper 1 style · no calculator**

The function f is defined by $f(x) = \frac{3x-1}{x+2}$ for $x \neq -2$. Find an expression for $f^{-1}(x)$.

2. **4 marks · Paper 2 style · calculator**

Solve the equation $\dfrac{x+1}{x-2} = x$. **[4]**

3. **4 marks · Paper 1 style · no calculator**

The graph of $y = \dfrac{px + q}{x + r}$ crosses the y-axis at $(0, 3)$. It has a vertical asymptote $x = -2$ and a horizontal asymptote $y = 4$.

Find the values of p, q and r. **[4]**

4. **4 marks · Paper 2 style · calculator**

Consider the rational function $g(x) = \frac{4x-5}{2x+3}$.

1. Use your graphic display calculator to find the exact coordinates of the x-intercept and the y-intercept.
2. Write down the equations of the horizontal and vertical asymptotes.

5. **4 marks · Paper 1 style · no calculator**

The graph of $f(x) = \dfrac{ax + 1}{x - b}$ has a vertical asymptote $x = 4$ and a horizontal asymptote $y = -2$.

(a) Write down the value of a and the value of b. **[2]**

(b) Find $f(5)$. **[2]**

6. **5 marks · Paper 1 style · no calculator**

A rational function is defined by $f(x) = \frac{ax+b}{cx-2}$. The graph of $f(x)$ has a vertical asymptote at $x = 1$, a horizontal asymptote at $y = 3$, and a y-intercept at $(0, -2.5)$.

Find the exact values of the constants a, b, and c.

7. **5 marks · Paper 1 style · no calculator**

Consider the function $f(x) = \frac{5x-2}{x+3}$, where $x \neq -3$.

1. Find an expression for the inverse function, $f^{-1}(x)$.
2. State the equations of the vertical and horizontal asymptotes of $f^{-1}(x)$.

8. **6 marks · Paper 1 style · no calculator**

Let $g(x) = \dfrac{3}{x+2} - 1$, for $x \neq -2$.

(a) Write down the equations of the asymptotes of the graph of g. **[2]**

(b) Find the x-intercept. **[2]**

(c) Describe a sequence of transformations that maps the graph of $y = \dfrac{1}{x}$ onto the graph of g. **[2]**

9. 6 marks · Paper 1 style · no calculator

(a) Solve $|2x - 5| = 3$. **[2]**

(b) Solve $|x - 1| = |2x + 4|$. **[4]**

10. 5 marks · Paper 1 style · no calculator

Let $f(x) = \dfrac{2x + 3}{x - 2}$, for $x \neq 2$.

(a) Show that $f^{-1}(x) = f(x)$. **[4]**

(b) Hence write down $(f \circ f)(7)$. **[1]**

11. 6 marks · Paper 1 style · no calculator

Let $f(x) = \left|\frac{6x-3}{2x-9}\right|$ for $x \neq 4.5$.

1. Write down the equations of the vertical and horizontal asymptotes of the graph of $f(x)$.
2. The horizontal line $y = k$ intersects the graph of $f(x)$ exactly once. Find the two possible exact values of k.

Answers and worked solutions

Question 1

4 marks

Answer: $f^{-}1(x) = (-2x-1)/(x-3)$

Let $y = f(x)$ and swap x and y: $x = \frac{3y-1}{y+2}$. **M1**

Multiply by the denominator:

$x(y+2) = 3y-1 \implies xy+2x = 3y-1$. **M1**

Collect the y terms: $xy-3y = -2x-1$. **M1**

Factorise and divide:

$y(x-3) = -2x-1 \implies f^{-1}(x) = \frac{-2x-1}{x-3}$ (or $\frac{2x+1}{3-x}$). **A1**

Question 2

4 marks

Answer: $x = 3.30$ or $x = -0.303$

$x+1 = x(x-2)$ **M1**

$x^2-3x-1 = 0$ **A1**

$x = 3.30$ or $x = -0.303$ **A1A1**

Question 3

4 marks

Answer: $p = 4,\ q = 6,\ r = 2$

$r = 2$ **A1**

$p = 4$ **A1**

$\frac{q}{2} = 3$ **M1**

$q = 6$ **A1**

Question 4

4 marks

Answer: (a) $(5/4, 0), (0, -5/3)$; (b) $x = -1.5, y = 2$

1. **CG50:** `MENU 5 (Graph)`. `Y1 = (4x-5)/(2x+3)`. `DRAW`.
   `G-Solv (F5) -> ROOT (F1)` gives $x = 1.25$. Coordinates: $\left(\frac{5}{4}, 0\right)$. **A1**
   `G-Solv -> Y-ICEPT (F4)` gives $y = -1.666\ldots$. Coordinates: $\left(0, -\frac{5}{3}\right)$ exactly. **A1**
2. Vertical asymptote (denominator zero): $2x+3 = 0 \implies x = -1.5$. **A1**
   Horizontal asymptote (ratio of leading coefficients): $y = \frac{4}{2} = 2$. **A1**

Question 5

4 marks

Answer: (a) $a = -2,\ b = 4$; (b) -9

(a)

$a = -2$ **A1**

$b = 4$ **A1**

(b)

$\frac{-10+1}{5-4}$ **(M1)**

-9 **A1**

Question 6

5 marks

Answer: $a = 6$; $b = 5$; $c = 2$

Vertical asymptote at $x = 1$, so the denominator is zero there: $c(1) - 2 = 0$. **M1**

$c = 2$. **A1**

Horizontal asymptote is the ratio of leading coefficients: $\frac{a}{c} = 3 \implies \frac{a}{2} = 3$. **M1**

$a = 6$. **A1**

Use the $y-$intercept:

$f(0) = \frac{b}{-2} = -2.5 \implies b = 5$. **A1**

Question 7

5 marks

Answer: (a) $f^{-}1(x) = (-3x-2)/(x-5)$; (b) $x = 5, y = -3$

1. Let $y = \frac{5x-2}{x+3}$. Swap x and y: $x = \frac{5y-2}{y+3}$. **M1**
   Multiply out: $xy+3x = 5y-2$.
   Collect y terms:
   $xy-5y = -3x-2 \implies y(x-5) = -3x-2$. **(A1)**
   $f^{-1}(x) = \frac{-3x-2}{x-5}$ (or $\frac{3x+2}{5-x}$). **A1**
2. Vertical asymptote: $x = 5$. **A1**
   Horizontal asymptote: $y = -3$. **A1**
   (Note: these are the asymptotes $y = 5$ and $x = -3$ of $f(x)$, reflected in $y = x$.)

Question 8

6 marks

Answer: (a) $x = -2$, $y = -1$; (b) 1

(a)

$x = -2$ **A1**

$y = -1$ **A1**

(b)

$\dfrac{3}{x+2} = 1$ **(M1)**

$x = 1$ **A1**

(c)

vertical stretch, scale factor 3 **A1**

translation by $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ **A1**

Question 9

6 marks

Answer: (a) $x = 4$ or $x = 1$; (b) $x = -1$ or $x = -5$

(a)

$2x - 5 = 3$ or $2x - 5 = -3$ **(M1)**

$x = 4$ or $x = 1$ **A1**

(b)

$(x-1)^2 = (2x+4)^2$ or two linear cases **M1**

$3x^2 + 18x + 15 = 0$ **A1**

$x = -1$ **A1**

$x = -5$ **A1**

Question 10

5 marks

Answer: (b) 7

(a)

$x = \dfrac{2y+3}{y-2}$ **M1**

$xy - 2x = 2y + 3$ **A1**

$y(x-2) = 2x + 3$ **A1**

$y = \dfrac{2x+3}{x-2}$, so $f^{-1} = f$ **A1 AG**

(b)

7 **A1**

Question 11

6 marks

Answer: (a) $x = 4.5, y = 3$; (b) $k = 0, k = 3$

1. Vertical asymptote (denominator zero): $x = 4.5$. **A1**
   Horizontal asymptote: $y = \left|\frac{6}{2}\right| = 3$. **A1**
2. Let $h(x) = \frac{6x-3}{2x-9} = 3 + \frac{24}{2x-9}$; h takes every real value except 3 exactly once, so $f(x) = k$ ($k > 0$) has two solutions ($h = k$ and $h = -k$) unless one of these is impossible. **R1**
   $k = 0$: $6x - 3 = 0 \implies x = 0.5$, a single intersection, so $k = 0$. **A1**
   $k = 3$: $h(x) = 3$ has no solution ($6x - 3 = 6x - 27$ is impossible), so solve $\frac{6x-3}{2x-9} = -3$. **M1**
   $6x - 3 = -6x + 27 \implies x = 2.5$, a single intersection, so $k = 3$. **A1**

Every solution is also on ibmathrevision.com/slaa/rational-and-reciprocal-functions-exam-practice
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