IB MATHS AA SL · UNIT 3

Radians: exam practice

10 questions · 45 marks · worked solutions and markschemes

Name _____ Date _____ Score / 45

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/radians-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Convert $150°$ to radians, giving your answer as an exact multiple of π.

2. **3 marks · Paper 1 style · no calculator**

A sector of a circle has radius r cm and a central angle of θ radians. The area of the sector is exactly 5 times the length of its arc. Find the exact value of r.

3. **5 marks · Paper 1 style · no calculator**

Consider a sector of a circle with radius R cm and an angle of θ radians at the centre. The area of the sector (in cm^2) is numerically equal to exactly three times the length of its arc (in cm). Find the exact value of R.

4. **5 marks · Paper 1 style · no calculator**

A sector of a circle with radius r cm and angle θ radians has an arc of length 15 cm and an area of $60\ \text{cm}^2$.

Find the value of r and the value of θ. **[5]**

5. **4 marks · Paper 1 style · no calculator**

(a) Convert $225°$ to radians, as an exact multiple of π. **[2]**

(b) Convert $\frac{7\pi}{9}$ radians to degrees. **[2]**

6. **5 marks · Paper 2 style · calculator**

A lawn sprinkler sprays water over a lawn covering an arc of 1.8 radians with a maximum spray distance of r metres. The lawn sprinkler perfectly waters a sector of area $20\ \text{m}^2$ of the lawn.

1. Calculate the value of the maximum spray distance, r.
2. Calculate the length of the outer arc watered by the sprinkler.

7. **5 marks · Paper 2 style · calculator**

A windscreen wiper blade is $0.8\ \text{m}$ long. When in motion, the blade moves through an arc of θ radians and wipes an area of $0.6\ \text{m}^2$ on the glass.

1. Calculate the value of θ.
2. Calculate the total distance travelled by the outer edge of the wiper blade during one single sweep.

8. **5 marks · Paper 1 style · no calculator**

A sector of a circle has radius 6 cm and angle $\frac{2\pi}{3}$ at the centre.

(a) Find the exact length of the arc. **[2]**

(b) Find the exact area of the sector. **[2]**

(c) Find the exact perimeter of the sector. **[1]**

9. **5 marks · Paper 2 style · calculator**

The chord $[AB]$ of a circle with centre O and radius 10 cm subtends an angle of 1.2 radians at O.

(a) Find the length of the chord $[AB]$. **[2]**

(b) Find the area of the minor segment cut off by the chord. **[3]**

10. **6 marks · Paper 1 style · no calculator**

A sector of a circle has radius r cm and a perimeter of 20 cm.

(a) Show that the area of the sector is $A = 10r - r^2$. **[3]**

(b) Find the maximum area of the sector and the angle at the centre when this happens. **[3]**

Answers and worked solutions

Question 1

2 marks

Answer: $5\pi/6$

$150 \times \frac{\pi}{180}$ (M1)

$\frac{5\pi}{6}$ A1

Question 2

3 marks

Answer: $r = 10$

Set area equal to 5 times arc length:

$\frac{1}{2}r^2\theta = 5(r\theta)$ M1

Divide both sides by $r\theta$ (since $r, \theta > 0$): $\frac{1}{2}r = 5$ M1

$r = 10$ A1

Question 3

5 marks

Answer: $R = 6$

Set up the expressions for area and arc length.

Area $= \frac{1}{2}R^2\theta$. (A1)

Arc length $= R\theta$. (A1)

Equating the area to three times the arc length:

$\frac{1}{2}R^2\theta = 3(R\theta)$. (M1)

Since $\theta > 0$ and $R > 0$, we can divide both sides by $R\theta$: (M1)

$\frac{1}{2}R = 3 \implies R = 6$. A1

Question 4

5 marks

Answer: $r = 8,\ \theta = \frac{15}{8}$

$r\theta = 15$ A1

$\frac{1}{2}r^2\theta = 60$ A1

$\frac{1}{2}r(r\theta) = 60$, so $\frac{15r}{2} = 60$ M1

$r = 8$ A1

$\theta = \frac{15}{8}$ A1

Question 5

4 marks

Answer: (a) $\frac{5\pi}{4}$; (b) $140°$

(a)

$225 \times \frac{\pi}{180}$ (M1)

$\frac{5\pi}{4}$ A1

(b)

$\frac{7\pi}{9} \times \frac{180}{\pi}$ (M1)

$140°$ A1

Question 6

5 marks

Answer: (a) $r = 4.71$; (b) $8.49\ m$

1. Area of a sector $= \frac{1}{2}r^2\theta$. M1
   $20 = \frac{1}{2}r^2(1.8) \implies 20 = 0.9r^2$. A1
   $r^2 = \frac{20}{0.9} = 22.222\ldots \implies r = \sqrt{22.222\ldots}$.
   $r = 4.714\ldots \implies 4.71\ \text{m}\ (3sf)$. A1
2. Arc length $l = r\theta$. (M1)
   $l = 4.714\ldots \times 1.8 = 8.485\ldots \implies 8.49\ \text{m}\ (3sf)$. A1

Question 7

5 marks

Answer: (a) $\theta = 1.875$; (b) $1.5\ m$

1. Area of wiped sector $= \frac{1}{2}r^2\theta$. (M1)
   $0.6 = \frac{1}{2}(0.8)^2\theta \implies 0.6 = 0.32\theta$. (A1)
   $\theta = \frac{0.6}{0.32} = 1.875$ radians. A1
2. The outer edge travels along the arc of the sector.
   Arc length $l = r\theta$. (M1)
   $l = 0.8 \times 1.875 = 1.5\ \text{m}$. A1

Question 8

5 marks

Answer: (a) 4π cm; (b) $12\pi\ \text{cm}^2$; (c) $4\pi + 12$ cm

(a)

$6 \times \frac{2\pi}{3}$ (M1)

4π cm A1

(b)

$\frac{1}{2} \times 36 \times \frac{2\pi}{3}$ (M1)

$12\pi \text{ cm}^2$ **A1**

(c)

$4\pi + 12$ cm **A1**

Question 9

5 marks

Answer: (a) 11.3 cm; (b) 13.4 cm^2

(a)

$AB^2 = 10^2 + 10^2 - 2(10)(10)\cos 1.2$ or $2 \times 10 \sin 0.6$ **(M1)**

11.3 cm **A1**

(b)

sector $\frac{1}{2}(10)^2(1.2) = 60$ **(A1)**

triangle $\frac{1}{2}(10)^2 \sin 1.2 = 46.60\ldots$ **(A1)**

segment 13.4 cm^2 **A1**

Question 10

6 marks

Answer: (b) 25 cm^2, $\theta = 2$

(a)

$2r + r\theta = 20$, so $\theta = \frac{20 - 2r}{r}$ $M1$**A1**

$A = \frac{1}{2}r^2 \times \frac{20 - 2r}{r} = 10r - r^2$ **A1 AG**

(b)

$A = 25 - (r - 5)^2$ or $A'(r) = 10 - 2r = 0$ **(M1)**

maximum area 25 cm^2 when $r = 5$ **A1**

$\theta = 2$ radians **A1**

Every solution is also on ibmathrevision.com/slaa/radians-exam-practice
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