IB MATHS AA SL · UNIT 2

Quadratic Functions: exam practice

10 questions · 44 marks · worked solutions and markschemes

Name _____ Date _____ Score / 44

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/quadratic-functions-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Solve $x^2 - 5x + 6 = 0$.

2. **4 marks · Paper 1 style · no calculator**

The quadratic function $f(x) = -2x^2 + 8x + c$ has a maximum value of 15. Find the exact value of the constant c.

3. **5 marks · Paper 1 style · no calculator**

The equation $kx^2 + 4x + k - 3 = 0$, where $k \neq 0$, has two distinct real roots.

Find the possible values of k. **[5]**

4. **6 marks · Paper 1 style · no calculator**

A straight line $y = mx - 4$ is tangent to the quadratic curve $y = x^2 - 2x + 5$. Find the possible values of m.

5. **4 marks · Paper 2 style · calculator**

Consider the quadratic function $g(x) = 3x^2 - 7.2x - 11.5$.

1. Use your graphic display calculator to find the coordinates of the x-intercepts, giving your answers to three significant figures.
2. Find the coordinates of the local minimum point on the graph of g.

6. **4 marks · Paper 1 style · no calculator**

Let $f(x) = 2x^2 - 12x + 11$.

(a) Write $f(x)$ in the form $a(x - h)^2 + k$. **[3]**

(b) Write down the coordinates of the vertex. **[1]**

7. 4 marks · Paper 2 style · calculator

A ball is thrown upwards. Its height, h metres, after t seconds is $h(t) = -4.9t^2 + 19.6t + 2$.

(a) Find the maximum height of the ball. **[2]**

(b) Find the time when the ball hits the ground. **[2]**

8. 5 marks · Paper 1 style · no calculator

A quadratic function f has x-intercepts at $x = 2$ and $x = -5$, and its graph passes through $(0, 20)$.

(a) Find $f(x)$ in the form $ax^2 + bx + c$. **[3]**

(b) Find the maximum value of $f(x)$. **[2]**

9. 5 marks · Paper 1 style · no calculator

(a) Solve $3x^2 - 5x - 2 = 0$. **[2]**

(b) Hence solve $3e^{2x} - 5e^x - 2 = 0$. **[3]**

10. **5 marks · Paper 1 style · no calculator**

Consider the equation $x^2 + (k+2)x + 2k = 0$.

(a) Show that the equation has real roots for every value of k. **[3]**

(b) Find the value of k for which the equation has equal roots, and find this root. **[2]**

Answers and worked solutions

Question 1

2 marks

Answer: $x = 2, x = 3$

$(x-2)(x-3) = 0$ (M1)

$x = 2,\ x = 3$ A1

Question 2

4 marks

Answer: $c = 7$

The maximum occurs at the vertex:

$x = -\frac{b}{2a} = -\frac{8}{2(-2)}$. M1

$x = 2$. A1

Set the maximum value:

$f(2) = -2(2)^2 + 8(2) + c = 15$. M1

$8 + c = 15 \implies c = 7$. A1

Question 3

5 marks

Answer: $-1 < k < 4,\ k \neq 0$

$\Delta = 16 - 4k(k-3) > 0$ M1

$k^2 - 3k - 4 < 0$ A1

$(k-4)(k+1) < 0$, critical values -1 and 4 A1

$-1 < k < 4$, $k \neq 0$ A1A1

Question 4

6 marks

Answer: $m = -8, m = 4$

Equate the line and the curve:

$x^2 - 2x + 5 = mx - 4$. M1

Rearrange: $x^2 - (m+2)x + 9 = 0$. M1

Tangent, so the discriminant is zero:

$(m+2)^2 - 4(1)(9) = 0$. M1

Expand: $m^2 + 4m - 32 = 0$. A1

Factorise: $(m+8)(m-4) = 0$. M1

$m = -8$ or $m = 4$ A1

Question 5

4 marks

Answer: (a) $(3.50, 0), (-1.10, 0)$; (b) $(1.2, -15.82)$

1. **CG50:** `MENU A (Equation) -> F2 (Polynomial) -> F1 (Degree 2)`. Input `a = 3, b = -7.2, c = -11.5`. Press EXE.
   $x_1 = 3.4963\ldots \implies x = 3.50$ (3 s.f.); coordinates $(3.50, 0)$. A1
   $x_2 = -1.0963\ldots \implies x = -1.10$ (3 s.f.); coordinates $(-1.10, 0)$. A1
2. **CG50:** `MENU 5 (Graph)`. `Y1 = 3x^2 - 7.2x - 11.5`. `DRAW`. Press `G-Solv (F5) -> MIN (F3)`. (M1)
   Coordinates: $(1.2, -15.82)$ $(= (1.2, -15.8))$. A1

Question 6

4 marks

Answer: (a) $2(x-3)^2 - 7$; (b) $(3, -7)$

(a)

$2(x^2 - 6x) + 11$ (M1)

$2(x-3)^2 - 18 + 11$ (A1)

$2(x-3)^2 - 7$ A1

(b)

$(3, -7)$ A1

Question 7

4 marks

Answer: (a) $21.6\ m$; (b) $t = 4.10\ s$

(a)

$t = 2$ (M1)

$21.6\ m$ A1

(b)

$-4.9t^2 + 19.6t + 2 = 0$ (M1)

$t = 4.10\ s$ A1

Question 8

5 marks

Answer: (a) $-2x^2 - 6x + 20$; (b) 24.5

(a)

$f(x) = a(x-2)(x+5)$ (M1)

$-10a = 20$, $a = -2$ A1

$f(x) = -2x^2 - 6x + 20$ A1

(b)

$x = -1.5$ **(M1)**

24.5 **A1**

Question 9

5 marks

Answer: (a) $x = 2$ or $x = -\frac{1}{3}$; (b) $x = \ln 2$

(a)

$(3x + 1)(x - 2) = 0$ **(M1)**

$x = 2,\ x = -\frac{1}{3}$ **A1**

(b)

$e^x = 2$ or $e^x = -\frac{1}{3}$ **M1**

$e^x > 0$, so reject $e^x = -\frac{1}{3}$ **R1**

$x = \ln 2$ **A1**

Question 10

5 marks

Answer: (b) $k = 2$, $x = -2$

(a)

$\Delta = (k + 2)^2 - 8k$ **M1**

$= k^2 - 4k + 4 = (k - 2)^2$ **A1**

$(k - 2)^2 \geq 0$ for all k, so the roots are real **R1**

(b)

$k = 2$ **A1**

$x^2 + 4x + 4 = 0$, $x = -2$ **A1**

Every solution is also on ibmathrevision.com/slaa/quadratic-functions-exam-practice
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