# Proof: exam practice

10 questions · 38 marks · worked solutions and markschemes

Name _____ Date _____ Score / 38

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/proof-exam-practice

**1.** **3 marks · Paper 1 style · no calculator**

Prove that the sum of any three consecutive integers is a multiple of $3$. **[3]**

**2.** **3 marks · Paper 1 style · no calculator**

Show that $(2n+3)^2-(2n-3)^2$ is a multiple of $12$ for every integer $n$. **[3]**

**3.** **3 marks · Paper 1 style · no calculator**

Prove that the product of an odd integer and an even integer is always even. **[3]**

**4.** **3 marks · Paper 1 style · no calculator**

Prove that $\dfrac{x^2-9}{x^2+x-6} \equiv \dfrac{x-3}{x-2}$, for $x \neq 2$, $x \neq -3$. **[3]**

**5.** **4 marks · Paper 1 style · no calculator**

Show that $\dfrac{2}{\sqrt{5}-\sqrt{3}} + \dfrac{2}{\sqrt{5}+\sqrt{3}} = 2\sqrt{5}$. **[4]**

**6.** **5 marks · Paper 1 style · no calculator**

Show that $\dfrac{1}{n} - \dfrac{1}{n+1} = \dfrac{1}{n(n+1)}$, and hence find the exact value of $\displaystyle\sum_{n=1}^{99} \frac{1}{n(n+1)}$. **[5]**

**7.** **4 marks · Paper 1 style · no calculator**

Prove that the difference between the squares of two consecutive odd integers is always a multiple of $8$. **[4]**

**8.** 4 marks · Paper 1 style · no calculator

Prove that $\frac{x}{y} + \frac{y}{x} \geq 2$ for all positive real numbers $x$ and $y$. **[4]**

**9.** 4 marks · Paper 1 style · no calculator

A student claims that $n^2 + n + 11$ is prime for every positive integer $n$.

**(a)** Show that the claim is true for $n = 1$ and $n = 2$. **[2]**

**(b)** By choosing a suitable value of $n$, show that the claim is false. **[2]**

**10.** 5 marks · Paper 1 style · no calculator

Let $S_n = \sum_{r=1}^{n} (2r - 1)$.

**(a)** Write down $S_1$, $S_2$ and $S_3$. **[2]**

**(b)** Prove that $S_n = n^2$. **[3]**

# Answers and worked solutions

## Question 1

3 marks

**Answer:** a proof: the worked solution above is the full answer

Let the integers be $m,\ m+1,\ m+2$ where $m \in \mathbb{Z}$ M1

Sum $= 3m+3$ A1

$= 3(m+1)$, and $m+1$ is an integer, so the sum is a multiple of $3$ R1

## Question 2

3 marks

**Answer:** a proof: the worked solution above is the full answer

Expanding: $(4n^2+12n+9)-(4n^2-12n+9)$ M1

$= 24n$ A1

$= 12(2n)$, and $2n$ is an integer, so it is a multiple of $12$ R1

## Question 3

3 marks

**Answer:** a proof: the worked solution above is the full answer

Odd: $2p+1$, even: $2q$, with $p, q \in \mathbb{Z}$ M1

Product $= 4pq+2q = 2(2pq+q)$ A1

$2pq+q$ is an integer, so the product is even R1

## Question 4

3 marks

**Answer:** a proof: the worked solution above is the full answer

$x^2-9 = (x-3)(x+3)$ A1

$x^2+x-6 = (x+3)(x-2)$ A1

cancel the common factor $(x+3)$, which is not zero, to get $\dfrac{x-3}{x-2}$ R1

## Question 5

4 marks

**Answer:** a proof: the worked solution above is the full answer

Common denominator $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3}) = 5-3 = 2$ $M1$A1

Numerator $2(\sqrt{5}+\sqrt{3})+2(\sqrt{5}-\sqrt{3}) = 4\sqrt{5}$ A1

$\dfrac{4\sqrt{5}}{2} = 2\sqrt{5}$ R1 AG

## Question 6

5 marks

**Answer:** $\frac{99}{100}$

$\dfrac{(n+1)-n}{n(n+1)}$ M1

$= \dfrac{1}{n(n+1)}$ AG

Sum $= \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{99}-\frac{1}{100}\right)$ M1

middle terms cancel, leaving $1-\dfrac{1}{100}$ A1

$= \dfrac{99}{100}$ A1A1

## Question 7

4 marks

**Answer:** a proof: the worked solution above is the full answer

Let the odd integers be $2m-1$ and $2m+1$, $m \in \mathbb{Z}$ M1

$(2m+1)^2-(2m-1)^2 = (4m^2+4m+1)-(4m^2-4m+1)$ A1

$= 8m$ A1

$m$ is an integer, so $8m$ is a multiple of $8$ R1

## Question 8

4 marks

**Answer:** a proof: the worked solution above is the full answer

Multiply by $xy > 0$: $x^2+y^2 \geq 2xy$ M1

equivalent to $x^2-2xy+y^2 \geq 0$ A1

$(x-y)^2 \geq 0$ A1

true for all real $x, y$, and each step reverses because $xy > 0$, so the inequality holds R1

## Question 9

4 marks

**Answer:** a proof: the worked solution above is the full answer

**(a)**

$n = 1$: $13$, prime A1

$n = 2$: $17$, prime A1

**(b)**

$n = 10$: $100+10+11 = 121$ M1

$121 = 11 \times 11$, which is not prime, so the claim is false A1

## Question 10

5 marks

**Answer:** (a) $S_1 = 1,\ S_2 = 4,\ S_3 = 9$

**(a)**

$1,\ 4,\ 9$ A2

**(b)**

arithmetic series with $u_1 = 1$, $d = 2$ M1

$S_n = \dfrac{n}{2}(2+2(n-1))$ A1

$= \dfrac{n}{2}(2n) = n^2$ A1 AG

Every solution is also on ibmathrevision.com/slaa/proof-exam-practice

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