IB MATHS AA SL · UNIT 4

Probability: exam practice

10 questions · 49 marks · worked solutions and markschemes

Name _____ Date _____ Score / 49

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/probability-exam-practice

1. **3 marks · Paper 1 style · no calculator**

A fair six-sided die is rolled twice. Find the exact probability that the sum of the two scores is exactly 8.

2. **5 marks · Paper 1 style · no calculator**

Events A and B are such that $P(A) = 0.45$, $P(A|B) = 0.3$, and $P(A \cup B) = 0.65$.

Find the exact value of $P(B)$.

3. **4 marks · Paper 2 style · calculator**

A biased coin has a probability of landing on tails of 0.38. The coin is tossed exactly 250 times.

1. Find the probability that the coin lands on heads on any single toss.
2. Calculate the expected number of times the coin will land on tails.
3. Calculate the expected number of times the coin will land on heads.

4. **4 marks · Paper 1 style · no calculator**

Events A and B are independent, with $P(A) = 0.4$ and $P(B) = 0.5$.

(a) Find $P(A \cap B)$. **[1]**

(b) Find $P(A \cup B)$. **[2]**

(c) Write down $P(A \mid B')$. **[1]**

5. **5 marks · Paper 2 style · calculator**

Two events C and D are such that $P(C) = 0.55$ and $P(D) = 0.4$. The probability of neither event occurring is $P(C' \cap D') = 0.27$.

1. Find $P(C \cup D)$.
2. Find $P(C \cap D)$.
3. Find the probability that event C occurs, but event D does not.

6. **5 marks · Paper 1 style · no calculator**

Events A and B have $P(A) = 0.3$, $P(B) = k$ and $P(A \cup B) = 0.6$.

(a) Find k if A and B are mutually exclusive. **[2]**

(b) Find k if A and B are independent. **[3]**

7. **6 marks · Paper 1 style · no calculator**

A bag contains 5 red counters and 3 blue counters. Two counters are taken at random, without replacement.

(a) Find the probability that both counters are red. **[2]**

(b) Find the probability that the counters are different colours. **[2]**

(c) Given that the counters are different colours, find the probability that the first one was red. **[2]**

8. **6 marks · Paper 1 style · no calculator**

In a class of 40 students, 22 play football, 15 play tennis and 8 play both.

(a) Find the number of students who play neither sport. **[2]**

(b) A student is chosen at random. Find the probability that they play football but not tennis. **[2]**

(c) A student who does not play tennis is chosen at random. Find the probability that they play football. **[2]**

9. **5 marks · Paper 1 style · no calculator**

A test for a condition gives a positive result for 95% of people who have the condition and for 4% of people who do not. 2% of the population have the condition.

(a) Find the probability that a randomly chosen person tests positive. **[3]**

(b) Given that a person tests positive, find the probability that they have the condition. **[2]**

10. **6 marks · Paper 1 style · no calculator**

For two events A and B, it is given that $n(A \cap B) = 5$, $n(A) = 11$, $n(A \cup B) = 12$, and $n(B') = 8$. Let $n(U)$ represent the total number of elements in the universal set.

1. Determine $n(U)$.
2. Find the exact probability $P(A \cup B)$.

Answers and worked solutions

Question 1

3 marks

Answer: $5/36$

1. Total number of equally likely outcomes: $6 \times 6 = 36$. (M1)
2. List the outcomes with sum 8: $(2,6), (3,5), (4,4), (5,3), (6,2)$, so 5 outcomes. M1
3. $P(\text{sum} = 8) = \frac{5}{36}$. A1

Question 2

5 marks

Answer: $2/7$

Addition rule:

$0.65 = 0.45 + P(B) - P(A \cap B) \implies P(B) - P(A \cap B) = 0.20$ M1

Conditional probability:

$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = 0.3$ M1

Rearrange: $P(A \cap B) = 0.3P(B)$ (A1)

Substitute into the first equation:

$P(B) - 0.3P(B) = 0.20 \implies 0.7P(B) = 0.20$ M1

$P(B) = \frac{2}{7}$ A1

Question 3

4 marks

Answer: (a) 0.62; (b) 95; (c) 155

1. Using complementary events: $P(H) = 1 - P(T) = 1 - 0.38 = 0.62$. A1
2. Expected occurrences $= n \times P(T)$. (M1)
   Expected tails $= 250 \times 0.38 = 95$. A1
3. Expected heads $= 250 \times 0.62 = 155$. A1

Question 4

4 marks

Answer: (a) 0.2; (b) 0.7; (c) 0.4

(a)

$0.4 \times 0.5 = 0.2$ A1

(b)

$0.4 + 0.5 - 0.2$ (M1)

0.7 A1

(c)

0.4 (independent) A1

Question 5

5 marks

Answer: (a) 0.73; (b) 0.22; (c) 0.33

1. $P(C' \cap D') = P((C \cup D)') = 0.27$, so $P(C \cup D) = 1 - 0.27 = 0.73$. A1
2. $P(C \cup D) = P(C) + P(D) - P(C \cap D)$. (M1)

   $0.73 = 0.55 + 0.4 - P(C \cap D) \implies P(C \cap D) = 0.95 - 0.73 = 0.22$. A1
3. $P(C \cap D') = P(C) - P(C \cap D)$. (M1)
   $= 0.55 - 0.22 = 0.33$. A1

Question 6

5 marks

Answer: (a) $k = 0.3$; (b) $\frac{3}{7}$

(a)

$0.3 + k = 0.6$ (M1)

$k = 0.3$ A1

(b)

$0.3 + k - 0.3k = 0.6$ $M1$A1

$k = \frac{3}{7}$ A1

Question 7

6 marks

Answer: (a) $\frac{5}{14}$; (b) $\frac{15}{28}$; (c) $\frac{1}{2}$

(a)

$\frac{5}{8} \times \frac{4}{7}$ (M1)

$\frac{5}{14}$ A1

(b)

$\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$ (M1)

$\frac{15}{28}$ A1

(c)

$\frac{15/56}{15/28}$ (M1)

$\frac{1}{2}$ **A1**

Every solution is also on ibmathrevision.com/slaa/probability-exam-practice
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Question 8

6 marks

Answer: (a) 11; (b) $\frac{7}{20}$; (c) $\frac{14}{25}$

(a)

$22 + 15 - 8 = 29$ **(M1)**

11 **A1**

(b)

$22 - 8 = 14$ **(M1)**

$\frac{14}{40} = \frac{7}{20}$ **A1**

(c)

$\frac{14}{25}$ **A2**

Question 9

5 marks

Answer: (a) 0.0582; (b) 0.326

(a)

$0.02 \times 0.95 + 0.98 \times 0.04$ $M1$**A1**

0.0582 **A1**

(b)

$\frac{0.019}{0.0582}$ **(M1)**

0.326 **A1**

Question 10

6 marks

Answer: (a) 14; (b) $6/7$

1. $n(A \cap B') = n(A) - n(A \cap B) = 11 - 5 = 6$. **M1**
   $n(B') = n(A \cap B') + n(A' \cap B')$, so $8 = 6 + n(A' \cap B') \implies n(A' \cap B') = 2$. **M1**
   $n(U) = n(A \cup B) + n(A' \cap B')$ **M1**
   $n(U) = 12 + 2 = 14$. **A1**
2. $P(A \cup B) = \frac{n(A \cup B)}{n(U)} = \frac{12}{14}$ **M1**
   $= \frac{6}{7}$. **A1**