IB MATHS AA SL · UNIT 4

Normal Distribution: exam practice

10 questions · 51 marks · worked solutions and markschemes

Name _____ Date _____ Score / 51

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/normal-distribution-exam-practice

1. **3 marks · Paper 2 style · calculator**

The continuous random variable X is normally distributed such that $X \sim N(55, 6^2)$. Given that $P(X < k) = 0.85$, find the value of k correct to three significant figures.

2. **4 marks · Paper 2 style · calculator**

The continuous random variable X follows a normal distribution such that $X \sim N(\mu, \sigma^2)$. Given that $P(X < 45) = 0.15$ and the mean $\mu = 58$, find the value of the standard deviation σ, correct to three significant figures.

3. 4 marks · Paper 2 style · calculator

The random variable X is normally distributed with mean μ and standard deviation 4. It is known that $P(X > 50) = 0.2$.

Find the value of μ. **[4]**

4. 6 marks · Paper 2 style · calculator

The time, T minutes, that customers wait at a call centre is normally distributed. $P(T < 20) = 0.1$ and $P(T > 40) = 0.25$.

Find the mean and the standard deviation of T. **[6]**

5. 4 marks · Paper 2 style · calculator

The weight, W in grams, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim N(200, 1.75^2)$.

1. Find $P(W < 195)$.
2. The manufacturer produces a batch of 500 chocolate bars. Calculate the expected number of chocolate bars from this batch that will weigh more than 203 g.

6. **5 marks · Paper 1 style · no calculator**

The random variable X is normally distributed with mean 30. It is given that $P(X < 26) = 0.2$.

(a) Write down $P(X > 34)$. **[1]**

(b) Find $P(26 < X < 34)$. **[2]**

(c) Find $P(X < 34 \mid X > 26)$. **[2]**

7. **6 marks · Paper 2 style · calculator**

The heights of adults in a town are normally distributed with mean 165 cm and standard deviation 8 cm.

(a) Find the probability that an adult is taller than 175 cm. **[2]**

(b) Find the probability that an adult is between 155 cm and 170 cm tall. **[2]**

(c) Find the height that 90% of adults are shorter than. **[2]**

8. **5 marks · Paper 2 style · calculator**

The mass of a bag of flour is normally distributed with mean 500 g and standard deviation 4 g. A bag is rejected if its mass is less than 495 g.

(a) Find the probability that a bag is rejected. **[2]**

(b) Ten bags are chosen at random. Find the probability that exactly two of them are rejected. **[3]**

9. **6 marks · Paper 2 style · calculator**

A machine is used to fill cans of a particular brand of soft drink. The volume, V in ml, of soft drink in the cans is normally distributed with a mean of $330\ \text{ml}$ and a standard deviation of $\sigma\ \text{ml}$.

1. Given that exactly 15% of the cans contain more than $333.4\ \text{ml}$ of soft drink, find the value of σ.
2. Find $P(320 \leq V \leq 340)$.
3. Six cans of the soft drink are chosen at random. Find the probability that all six cans contain less than $329\ \text{ml}$ of soft drink.

10. **8 marks · Paper 2 style · calculator**

The running time of feature films at a cinema is normally distributed with a mean time of 102 minutes and a standard deviation of 13 minutes.

1. Find the probability that a randomly selected feature film has a running time of more than 99 minutes.
2. Over the course of a month, Jonah watches exactly 18 different feature films. Find the expected number of occasions on which the film he watches will last less than 95 minutes.
3. Find the probability that on at least 6 out of the 18 occasions, the film will last for longer than 99 minutes.

Answers and worked solutions

Question 1

3 marks

Answer: $k = 61.2$

Use inverse normal on the GDC **(M1)**
with area 0.85, $\mu = 55$, $\sigma = 6$ **(A1)**
$k = 61.2$ (3 s.f.) **A1**

Question 2

4 marks

Answer: $\sigma = 12.5$

1. Use inverse normal on the GDC for an area of 0.15 in the standard normal distribution. **M1**
2. $z = -1.0364\ldots$ **A1**
3. Set up the standardisation equation: $\frac{45-58}{\sigma} = -1.0364\ldots$ **M1**
4. $\sigma = \frac{-13}{-1.0364\ldots} = 12.543\ldots \approx 12.5$ (3 s.f.). **A1**

Question 3

4 marks

Answer: $\mu = 46.6$

$P(X < 50) = 0.8$ **(M1)**
$z = 0.8416\ldots$ **(A1)**
$\frac{50-\mu}{4} = 0.8416\ldots$ **M1**
$\mu = 46.6$ **A1**

Question 4

6 marks

Answer: $\mu = 33.1,\ \sigma = 10.2$

$\frac{20-\mu}{\sigma} = -1.2815\ldots$ $M1$**A1**
$\frac{40-\mu}{\sigma} = 0.6744\ldots$ **A1**
solving simultaneously **(M1)**
$\sigma = 10.2$ **A1**
$\mu = 33.1$ **A1**

Question 5

4 marks

Answer: (a) 0.00214; (b) 21.6

Let $W \sim N(200, 1.75^2)$.

1. **CG50:** `STAT -> DIST -> NORM -> Ncd`. $\text{Lower} = -1 \times 10^{99}$, $\text{Upper} = 195$, $\sigma = 1.75$, $\mu = 200$. **(M1)**
   $P(W < 195) = 0.002137\ldots \implies 0.00214\ (3sf)$. **A1**
2. We need $P(W > 203)$. **CG50:** `Ncd`. $\text{Lower} = 203$, $\text{Upper} = 1 \times 10^{99}$, $\sigma = 1.75$, $\mu = 200$.
   $P(W > 203) = 0.04323\ldots$
   Multiply by the batch size: expected number $= 500 \times 0.04323\ldots$. **(M1)**
   $= 21.619\ldots \implies 21.6$ chocolate bars $(3sf)$. **A1**
   (An expected value need not be a whole number.)

Question 6

5 marks

Answer: (a) 0.2; (b) 0.6; (c) 0.75

(a)

0.2 (by symmetry) **A1**

(b)

$1 - 0.2 - 0.2$ **(M1)**
0.6 **A1**

(c)

$\frac{0.6}{0.8}$ **(M1)**
0.75 **A1**

Question 7

6 marks

Answer: (a) 0.106; (b) 0.628; (c) 175 cm

(a)

$P(H > 175)$ **(M1)**
0.106 **A1**

(b)

$P(155 < H < 170)$ **(M1)**
0.628 **A1**

(c)

inverse normal, 0.9 **(M1)**

175 cm ($175.25\ldots$) **A1**

Question 8

5 marks

Answer: (a) 0.106; (b) 0.206

(a)

$P(M < 495)$ **(M1)**

0.106 **A1**

(b)

$X \sim B(10,\ 0.1056\ldots)$ **(M1)**

$\binom{10}{2}(0.1056\ldots)^2(0.8943\ldots)^8$ **(A1)**

0.206 **A1**

Question 9

6 marks

Answer: (a) $\sigma = 3.28$; (b) 0.998; (c) 0.00302

1. Given $P(V > 333.4) = 0.15 \implies P(V < 333.4) = 0.85$.
   Standardize: $P\left(Z < \frac{333.4-330}{\sigma}\right) = 0.85$. **(M1)**
   CG50: InvN (Area 0.85, $\mu = 0, \sigma = 1$)
   $\implies z = 1.0364\ldots$ **(A1)**
   $\frac{333.4-330}{\sigma} = 1.0364\ldots \implies \frac{3.4}{\sigma} = 1.0364\ldots \implies \sigma = 3.280\ldots \implies 3.28 \text{ ml}$. **A1**
2. **CG50:** Ncd with $\text{Lower} = 320$, $\text{Upper} = 340$, $\sigma = 3.2805\ldots$, $\mu = 330$.
   $P = 0.9976\ldots \implies 0.998\ (3sf)$. **A1**
3. Find $P(V < 329)$ for one can: Ncd
   $\implies P = 0.3802\ldots$ **(M1)**
   For six independent cans, multiply probabilities:
   $(0.3802\ldots)^6 = 0.003022\ldots \implies 0.00302\ (3sf)$. **A1**

Question 10

8 marks

Answer: (a) 0.591; (b) 5.31; (c) 0.993

1. Let $T \sim N(102, 13^2)$. Find $P(T > 99)$. **(M1)**
   CG50: Ncd ($\text{Lower} = 99, \text{Upper} = 1 \times 10^{99}, \sigma = 13, \mu = 102$).
   $P(T > 99) = 0.5912\ldots \implies 0.591\ (3sf)$. **A1**
2. Find $P(T < 95)$. **CG50:** Ncd ($\text{Lower} = -1 \times 10^{99}, \text{Upper} = 95, \sigma = 13, \mu = 102$). **(M1)**
   $P(T < 95) = 0.2951\ldots$ **(A1)**
   Expected number $= 18 \times 0.2951\ldots = 5.312\ldots \implies 5.31$ occasions (an expected value need not be a whole number). **A1**
3. Let Y be the number of occasions the film lasts longer than 99 mins.
   $Y \sim B(18, 0.5912\ldots)$. **(M1)**
   We need $P(Y \geq 6) = 1 - P(Y \leq 5)$. **(M1)**
   CG50: STAT -> DIST -> BINOMIAL -> Bcd with $\text{Lower} = 6, \text{Upper} = 18, \text{Numtrial} = 18, p = 0.5912\ldots$
   $P(Y \geq 6) = 0.99285\ldots \implies 0.993\ (3sf)$. **A1**

Every solution is also on ibmathrevision.com/slaa/normal-distribution-exam-practice
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