IB MATHS AA SL · UNIT 4

Measures of Central Tendency: exam practice

11 questions · 48 marks · worked solutions and markschemes

Name _____ Date _____ Score / 48

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/measures-of-central-tendency-exam-practice

1. **3 marks · Paper 1 style · no calculator**

The mean score of 10 students on a test is 22. An 11^{th} student takes the test, increasing the overall mean of the class to 23. Find the exact score of the 11^{th} student.

2. **4 marks · Paper 2 style · calculator**

There are two mathematics classes, Class P and Class Q, who take the same examination. Class P has 24 students and a mean score of 68%. Class Q has 16 students and a mean score of 75%. Find the overall mean score of all 40 students combined. Give your answer to one decimal place.

3. **4 marks · Paper 1 style · no calculator**

Five positive integers have a mode of 4, a median of 5, a mean of 6 and a range of 7.

Find the five integers. **[4]**

4. **4 marks · Paper 1 style · no calculator**

The number of goals scored by a football team in their last 7 matches are:

$$1, \quad 3, \quad 0, \quad 2, \quad 2, \quad 5, \quad 1$$

Find the exact values of:

1. the mode.
2. the median.
3. the mean.

5. **4 marks · Paper 2 style · calculator**

The shoe sizes of 25 students in a mathematics class are recorded in the frequency table below.

Shoe Size	**6**	**7**	**8**	**9**	**10**
Frequency	3	5	9	6	2

Use your graphic display calculator to find:

1. the mean shoe size.
2. the standard deviation.
3. the variance.

6. **4 marks · Paper 1 style · no calculator**

A dataset of exam scores has a mean of 42 and a standard deviation of 5. The teacher decides to adjust the grades by multiplying every score by 2, and then adding 10 to each score. Find the exact value of:

1. the new mean.
2. the new standard deviation.
3. the new variance.

7. **4 marks · Paper 1 style · no calculator**

The number of books read by 20 students last month is shown.

Books	0	1	2	3	4
Frequency	2	5	k	4	3

(a) Find the value of k. **[1]**

(b) Find the mean number of books. **[2]**

(c) Write down the median. **[1]**

8. **4 marks · Paper 1 style · no calculator**

The masses, m kg, of 40 parcels are grouped in the table.

Mass, m	$0 < m \leq 2$	$2 < m \leq 4$	$4 < m \leq 6$	$6 < m \leq 8$
Frequency	6	14	12	8

(a) Write down the modal class. **[1]**

(b) Use mid-interval values to estimate the mean mass. **[3]**

9. **5 marks · Paper 1 style · no calculator**

A dataset contains 5 numbers: $x, y, 4, 6, 8$. The mean of the dataset is 5 and the variance is 4. Given that $x < y$, find the exact values of x and y.

10. **6 marks · Paper 2 style · calculator**

The times, in minutes, that twelve students took to finish a puzzle are:

$$14,\ 17,\ 18,\ 21,\ 22,\ 22,\ 25,\ 27,\ 29,\ 30,\ 34,\ 52$$

(a) Find the mean and the median. **[2]**

(b) Find the lower quartile and the upper quartile. **[2]**

(c) Determine whether 52 is an outlier. **[2]**

11. **6 marks · Paper 2 style · calculator**

The ages of the members of a chess club are summarised by $n = 15$, $\sum x = 630$ and $\sum x^2 = 28\,350$.

(a) Find the mean age. **[1]**

(b) Find the standard deviation of the ages. **[3]**

(c) In five years' time, the same members are still in the club. Write down the new mean and the new standard deviation. **[2]**

Answers and worked solutions

Question 1

3 marks

Answer: 33

1. Sum of the first 10 scores: $10 \times 22 = 220$. M1
2. Sum of all 11 scores: $11 \times 23 = 253$. M1
3. Score of the 11^{th} student: $253 - 220 = 33$. A1

Question 2

4 marks

Answer: 70.8

Total score for Class $P = 24 \times 68 = 1632$. (M1)
Total score for Class $Q = 16 \times 75 = 1200$. (M1)
Total combined score $= 1632 + 1200 = 2832$. (A1)
Total number of students $= 24 + 16 = 40$.
Overall mean $= \frac{2832}{40} = 70.8\%$. A1

Question 3

4 marks

Answer: $4,\ 4,\ 5,\ 6,\ 11$

middle value 5; mode 4 so two values are 4 (M1)

smallest 4, so largest $4 + 7 = 11$ A1

sum $= 30$, so $4 + 4 + 5 + d + 11 = 30$ M1

$d = 6$, giving $4,\ 4,\ 5,\ 6,\ 11$ A1

Question 4

4 marks

Answer: (Mode) $1, 2$; (Median) 2; (Mean) 2

1. The mode is the most frequent value; 1 and 2 both occur twice.
   Mode $= 1$ and 2 (bimodal). A1
2. Order the data: $0, 1, 1, 2, 2, 3, 5$. (M1)
   The middle (4th) value is the median: 2. A1
3. Mean $= \frac{0+1+1+2+2+3+5}{7} = \frac{14}{7} = 2$. A1

Question 5

4 marks

Answer: (Mean) 7.96; (Standard deviation) 1.11; (Variance) 1.24

CG50: `MENU 2 (STAT)`.
Enter shoe sizes in `List 1` $\{6, 7, 8, 9, 10\}$.
Enter frequencies in `List 2` $\{3, 5, 9, 6, 2\}$.
`CALC (F2) -> SET (F6)`. Ensure `1Var XList = List1` and `1Var Freq = List2`. (M1)

1. Press `1-VAR (F1)`. $\bar{x} = 7.96$. A1
2. Read the standard deviation
   $\sigma_x = 1.11283\ldots \implies 1.11$ (3 s.f.). A1
3. Variance is the standard deviation squared:
   $\sigma^2 = (1.11283\ldots)^2 = 1.2384 \implies 1.24$ (3 s.f.). A1

Question 6

4 marks

Answer: (Mean) 94; (Standard deviation) 10; (Variance) 100

1. When every value is transformed by $y = ax + b$, the new mean is $a\mu + b$. (M1)
   New mean $= 2(42) + 10 = 94$. A1
2. Standard deviation is multiplied by $|a|$ but **not** affected by adding a constant.
   New SD $= 2 \times 5 = 10$. A1
3. Variance is the square of the standard deviation:
   New variance $= 10^2 = 100$. A1

Question 7

4 marks

Answer: (a) 6; (b) 2.05; (c) 2

(a)

$2 + 5 + k + 4 + 3 = 20$, $k = 6$ A1

(b)

$\frac{0 + 5 + 12 + 12 + 12}{20}$ (M1)

2.05 A1

(c)

2 A1

Question 8

4 marks

Answer: (b) 4.1 kg

(a)

$2 < m \leq 4$ **A1**

(b)

mid-values $1, 3, 5, 7$ **(A1)**

$\frac{6 + 42 + 60 + 56}{40}$ **(M1)**

4.1 kg **A1**

Question 9

5 marks

Answer: $x = 2$; $y = 5$

Use the mean to form an equation:

$\frac{x+y+18}{5} = 5 \implies x + y = 7$ **A1**

Use $\text{Var} = \frac{\sum x^2}{n} - \mu^2$:

$\frac{x^2+y^2+16+36+64}{5} - 25 = 4$ **M1**

Simplify:

$x^2 + y^2 + 116 = 145 \implies x^2 + y^2 = 29$ **A1**

Solve simultaneously, e.g.

$y = 7 - x \implies x^2 + (7 - x)^2 = 29 \implies x^2 - 7x + 10 = 0 \implies (x - 2)(x - 5) = 0$

M1

Since $x < y$: $x = 2$, $y = 5$ **A1**

Question 10

6 marks

Answer: (a) mean 25.9, median 23.5; (b) $Q_1 = 19.5,\ Q_3 = 29.5$

(a)

mean 25.9 **A1**

median 23.5 **A1**

(b)

$Q_1 = 19.5$ **A1**

$Q_3 = 29.5$ **A1**

(c)

$Q_3 + 1.5 \times \text{IQR} = 29.5 + 15 = 44.5$ **M1**

$52 > 44.5$, so it is an outlier **R1**

Question 11

6 marks

Answer: (a) 42; (b) $\sigma = 11.2$; (c) 47 and 11.2

(a)

$\frac{630}{15} = 42$ **A1**

(b)

$\sigma^2 = \frac{28\,350}{15} - 42^2$ **M1**

$= 126$ **(A1)**

$\sigma = 11.2$ **A1**

(c)

mean 47 **A1**

standard deviation unchanged, 11.2 **A1**

Every solution is also on ibmathrevision.com/slaa/measures-of-central-tendency-exam-practice
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