IB MATHS AA SL · UNIT 5

Limits and Rates of Change: exam practice

10 questions · 35 marks · worked solutions and markschemes

Name _____ Date _____ Score / 35

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/limits-and-rates-of-change-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Find $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$.

2. **2 marks · Paper 1 style · no calculator**

Find the average rate of change of $f(x) = x^2$ between $x = 1$ and $x = 3$.

3. **3 marks · Paper 1 style · no calculator**

Find $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$. **[3]**

4. **4 marks · Paper 1 style · no calculator**

Let $f(x) = x^2 - 3x$. Use differentiation from first principles to show that $f'(x) = 2x - 3$. **[4]**

5. **5 marks · Paper 1 style · no calculator**

Let $f(x) = 2x^3$. Use differentiation from first principles to find $f'(x)$. **[5]**

6. **3 marks · Paper 2 style · calculator**

Let $g(x) = \dfrac{\sin x}{x}$, where x is in radians.

(a) Find $g(0.1)$ and $g(0.01)$, giving your answers to 5 significant figures. **[2]**

(b) Hence write down the value of $\lim\limits_{x \to 0} g(x)$. **[1]**

7. 5 marks · Paper 1 style · no calculator

Let $f(x) = 4x^2 - x^3$.

(a) Find $f'(x)$. **[2]**

(b) Find the gradient of the graph at $x = 2$. **[1]**

(c) Find the values of x where the gradient is zero. **[2]**

8. 3 marks · Paper 2 style · calculator

Let $g(x) = \dfrac{e^x - 1}{x}$, $x \neq 0$.

a. Find $g(0.1)$, $g(0.01)$ and $g(0.001)$, correct to four significant figures. **[2]**

b. Hence write down the value of $\lim_{x \to 0} g(x)$. **[1]**

9. 4 marks · Paper 1 style · no calculator

Let $f(x) = x^3$.

(a) Show that the average rate of change of f between $x = 1$ and $x = 1 + h$ is $3 + 3h + h^2$. **[3]**

(b) Hence write down the gradient of the graph of f at $x = 1$. **[1]**

10. **4 marks · Paper 2 style · calculator**

The volume of water in a tank, V litres, t minutes after a tap is opened is $V(t) = 200 - 40\ln(t+1)$.

(a) Find the average rate of change of V over the first 3 minutes. **[2]**

(b) Find the instantaneous rate of change of V when $t = 3$. **[2]**

Answers and worked solutions

Question 1

2 marks

Answer: 6

$\frac{(x-3)(x+3)}{x-3} = x+3$ for $x \neq 3$ (M1)

limit $= 6$ A1

Question 2

2 marks

Answer: 4

$\frac{f(3)-f(1)}{3-1} = \frac{9-1}{2}$ (M1)

$= 4$ A1

Question 3

3 marks

Answer: $\frac{5}{4}$

$\frac{(x+3)(x-2)}{(x+2)(x-2)}$ $M1$A1

$\frac{5}{4}$ A1

Question 4

4 marks

Answer: a proof: the worked solution above is the full answer

$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - (x^2-3x)}{h}$

M1

$= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h}$ A1

$= \lim_{h\to 0}(2x + h - 3)$ A1

$= 2x - 3$ A1 AG

Question 5

5 marks

Answer: $6x^2$

$\frac{2(x+h)^3 - 2x^3}{h}$ M1

$2(x+h)^3 = 2x^3 + 6x^2h + 6xh^2 + 2h^3$ A1

$\frac{6x^2h + 6xh^2 + 2h^3}{h} = 6x^2 + 6xh + 2h^2$ A1

as $h \to 0$ M1

$f'(x) = 6x^2$ A1

Question 6

3 marks

Answer: (a) 0.99833 and 0.99998; (b) 1

(a)

0.99833 A1

0.99998 A1

(b)

1 A1

Question 7

5 marks

Answer: (a) $-3x^2 + 8x$; (b) 4; (c) $x = 0$ and $x = \frac{8}{3}$

(a)

$8x - 3x^2$ A1A1

(b)

$16 - 12 = 4$ A1

(c)

$x(8 - 3x) = 0$ (M1)

$x = 0,\ x = \frac{8}{3}$ A1

Question 8

3 marks

Answer: (a) $1.052, 1.005, 1.001$; (b) 1

a. $1.052,\ 1.005,\ 1.001$ A2

b. 1 A1

Question 9

4 marks

Answer: (b) 3

(a)

$\frac{(1+h)^3 - 1}{h}$ M1

$(1+h)^3 = 1 + 3h + 3h^2 + h^3$ A1

$\frac{3h + 3h^2 + h^3}{h} = 3 + 3h + h^2$ A1 AG

(b)

3 A1

Question 10

4 marks

Answer: (a) -18.5 litres per minute; (b) -10 litres per minute

(a)

$$\frac{V(3)-V(0)}{3}$$ **(M1)**

-18.5 litres per minute **A1**

(b)

$$V'(t)=-\frac{40}{t+1}$$ **(M1)**

-10 litres per minute **A1**

Every solution is also on ibmathrevision.com/slaa/limits-and-rates-of-change-exam-practice

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