IB MATHS AA SL · UNIT 5

Kinematics: exam practice

10 questions · 53 marks · worked solutions and markschemes

Name _____ Date _____ Score / 53

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/kinematics-exam-practice

1. **5 marks · Paper 2 style · calculator**

The population of fish in a lake is modelled by $P(t) = -0.2t^3 + 2.4t^2 + 10t + 50$, where t is the number of years since 2020 ($0 \leq t \leq 12$). Find the exact year during which the fish population is growing at its fastest rate.

2. **4 marks · Paper 2 style · calculator**

A cricket ball is projected directly upwards from ground level. The vertical height of the cricket ball, h metres above the ground, after t seconds is modelled by the function:

$$h(t) = 13t - 4.9t^2, \quad t \geq 0$$

1. Use your graphic display calculator to find the times at which the cricket ball is exactly 3 m above the ground.
2. Calculate the maximum height reached by the cricket ball.

3. 4 marks · Paper 2 style · calculator

The velocity of a cyclist who stops pedalling is $v(t) = 20e^{-0.4t}$ m s^{-1}, t seconds later.

(a) Find the distance travelled in the first 5 seconds. **[2]**

(b) Find the acceleration when $t = 2$. **[2]**

4. 5 marks · Paper 2 style · calculator

A particle moves with velocity $v(t) = 2t\sin t$ m s^{-1}, for $0 \leq t \leq 6$.

(a) Find the value of t, for $0 < t \leq 6$, when the particle is at rest. **[1]**

(b) Find the total distance travelled. **[2]**

(c) Find the acceleration when $t = 2$. **[2]**

5. 6 marks · Paper 1 style · no calculator

A particle moves in a straight line. Its displacement from a fixed point O after t seconds is $s(t) = t^3 - 6t^2 + 9t$ metres, for $t \geq 0$.

(a) Find the velocity at time t. **[1]**

(b) Find the times when the particle is at rest. **[2]**

(c) Find the total distance travelled in the first 3 seconds. **[3]**

6. **6 marks · Paper 2 style · calculator**

A particle moves with velocity $v(t) = e^{0.5t} - 3$ m s^{-1}, for $t \geq 0$.

(a) Find the exact time when the particle is at rest. **[2]**

(b) Find the displacement of the particle after 4 seconds. **[2]**

(c) Find the distance travelled in the first 4 seconds. **[2]**

7. **4 marks · Paper 2 style · calculator**

A particle moves with velocity $v(t) = t^2 - 5t + 4$ m s^{-1}, for $0 \leq t \leq 5$.

(a) Find the displacement of the particle after 5 seconds. **[2]**

(b) Find the total distance travelled. **[2]**

8. **6 marks · Paper 2 style · calculator**

A particle moves in a straight line with velocity $v(t) = 6t - t^2$ ms^{-1} for $0 \leq t \leq 8$ seconds.

1. Find the time at which the particle comes instantaneously to rest.
2. Calculate the total distance travelled by the particle during the first 8 seconds.

9. **6 marks · Paper 1 style · no calculator**

A particle moves along a straight line relative to a fixed point. The motion of the particle is modelled by the displacement function:

$$s(t) = \frac{t^3}{20} - \frac{17t^2}{8} + 18t - 4, \quad 0 \le t \le 30$$

1. Find an expression for the velocity of the particle, $v(t)$.
2. By setting your velocity expression to zero, find the exact times at which the particle is stationary.

10. **7 marks · Paper 1 style · no calculator**

A particle starts from O with velocity 3 m s^{-1}. Its acceleration is $a(t) = 6t - 4$ m s^{-2}.

(a) Find an expression for $v(t)$. **[2]**

(b) Show that the particle is never at rest. **[2]**

(c) Find the displacement from O when $t = 2$. **[3]**

Answers and worked solutions

Question 1

5 marks

Answer: 2024

Growth rate: $P'(t) = -0.6t^2 + 4.8t + 10$ A1

The growth rate is greatest where $P''(t) = 0$:

$P''(t) = -1.2t + 4.8 = 0$ M1

$t = 4$ A1

$P'''(t) = -1.2 < 0$ (or P' is a downward parabola), so this is a maximum of P' R1

Year: $2020 + 4 = 2024$ A1

Question 2

4 marks

Answer: (a) $t = 0.255, t = 2.40$; (b) $8.62\,m$

1. Set $h(t) = 3$:
   $13t - 4.9t^2 = 3 \implies -4.9t^2 + 13t - 3 = 0$ (M1)
   CG50: `Equation` → `Polynomial` → Degree 2.
   $t = 0.255$ s and $t = 2.40$ s (3 s.f.) A1
2. Maximum height when $v(t) = h'(t) = 0$:
   $13 - 9.8t = 0 \implies t = \frac{13}{9.8} = 1.3265\ldots$ s (M1)
   Substitute into $h(t)$ (or use `G-Solv -> MAX`):
   $h_{max} = 13(1.3265\ldots) - 4.9(1.3265\ldots)^2 = 8.62$ m (3 s.f.) A1

Question 3

4 marks

Answer: (a) $43.2\ m$; (b) $-3.59\ ms^{-2}$

(a)

$\int_0^5 20e^{-0.4t}\,dt$ (M1)

$43.2\ m$ A1

(b)

$a(t) = -8e^{-0.4t}$ (M1)

$-3.59\ ms^{-2}$ A1

Question 4

5 marks

Answer: (a) $t = \pi$; (b) $24.6\ m$; (c) $0.154\ ms^{-2}$

(a)

$t = \pi\ (3.14)$ A1

(b)

$\int_0^6 |2t \sin t|\,dt$ (M1)

$24.6\ m$ A1

(c)

$a = 2\sin t + 2t\cos t$ (M1)

$0.154\ ms^{-2}$ A1

Question 5

6 marks

Answer: (a) $3t^2 - 12t + 9$; (b) $t = 1$ and $t = 3$; (c) $8\ m$

(a)

$v(t) = 3t^2 - 12t + 9$ A1

(b)

$3(t-1)(t-3) = 0$ (M1)

$t = 1$ and $t = 3$ A1

(c)

$s(0) = 0$, $s(1) = 4$, $s(3) = 0$ A1

$4 + 4$ (M1)

$8\ m$ A1

Question 6

6 marks

Answer: (a) $t = 2\ln 3$; (b) $0.778\ m$; (c) $5.96\ m$

(a)

$e^{0.5t} = 3$ (M1)

$t = 2\ln 3\ (2.20)$ A1

(b)

$\int_0^4 (e^{0.5t} - 3)\,dt$ (M1)

$0.778\ m$ A1

(c)

$\int_0^4 |e^{0.5t} - 3|\,dt$ (M1)

$5.96\ m$ A1

Question 7

4 marks

Answer: (a) $-\frac{5}{6}\ m$; (b) $8.17\ m\ \left(\frac{49}{6}\right)$

(a)

$\int_0^5 v\,dt$ **(M1)**

$-\frac{5}{6}\ m\ (-0.833)$ **A1**

(b)

$\int_0^5 |v|\,dt$ **(M1)**

$\frac{49}{6} = 8.17\ m$ **A1**

Question 8

6 marks

Answer: (a) $t = 6$; (b) $152/3\ m$

1. The particle is at rest when $v(t) = 0$:
   $6t - t^2 = 0$ **M1**
   $t(6-t) = 0 \implies t = 6$ s (rejecting $t = 0$, the start) **A1**
2. Total distance $= \int_0^8 |6t - t^2|\,dt$ **M1**
   Split at $t = 6$, where v changes sign:
   $\int_0^6 (6t - t^2)\,dt + \int_6^8 (t^2 - 6t)\,dt$ **M1**
   $\left[3t^2 - \frac{t^3}{3}\right]_0^6 = 36$ and $\left[\frac{t^3}{3} - 3t^2\right]_6^8 = \left(\frac{512}{3} - 192\right) - (72 - 108) = \frac{44}{3}$ **A1**
   Total distance $= 36 + \frac{44}{3} = \frac{152}{3} = 50.7$ m (3 s.f.) **A1**

Question 9

6 marks

Answer: (a) $v(t) = 3t^2/20 - 17t/4 + 18$; (b) $t = (85 + \sqrt{2905})/6, t = (85 - \sqrt{2905})/6$

1. Differentiate $s(t)$ term by term:
   $v(t) = s'(t) = \frac{3t^2}{20} - 2\left(\frac{17t}{8}\right) + 18$. **(M1)**
   $v(t) = \frac{3t^2}{20} - \frac{17t}{4} + 18$. **A1**
2. Set $v(t) = 0$: $\frac{3t^2}{20} - \frac{17t}{4} + 18 = 0$. **(M1)**
   Multiply the entire equation by 20 to clear denominators: $3t^2 - 85t + 360 = 0$. **(A1)**
   The discriminant
   $b^2 - 4ac = 85^2 - 4(3)(360) = 7225 - 4320 = 2905$
   is not a perfect square, so use the quadratic formula:
   $t = \frac{85 \pm \sqrt{2905}}{6}$ **A1A1**
   (Both values, $t \approx 5.18$ and $t \approx 23.1$, lie in $0 \le t \le 30$.)

Question 10

7 marks

Answer: (a) $3t^2 - 4t + 3$; (c) $6\ m$

(a)

$v = 3t^2 - 4t + C$ **M1**

$v(t) = 3t^2 - 4t + 3$ **A1**

(b)

discriminant $16 - 36 < 0$ **M1**

so $v(t) \neq 0$ for all t **R1**

(c)

$s = t^3 - 2t^2 + 3t$ (as $s(0) = 0$) $M1$**A1**

$6\ m$ **A1**

Every solution is also on ibmathrevision.com/slaa/kinematics-exam-practice
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