IB MATHS AA SL · UNIT 5

Integration: exam practice

10 questions · 49 marks · worked solutions and markschemes

Name _____ Date _____ Score / 49

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/integration-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Find $\int (2x+1)^5 \, dx$. **[3]**

2. **3 marks · Paper 1 style · no calculator**

Find $\int \frac{x}{x^2+4} \, dx$. **[3]**

3. **4 marks · Paper 1 style · no calculator**

Find $\int \left(\sin 3x + e^{-2x}\right) dx$. **[4]**

4. **5 marks · Paper 1 style · no calculator**

The function f is such that $f'(x) = 3x^2 + \frac{2}{x^2}$ for $x > 0$, and $f(1) = 4$.

Find an expression for $f(x)$. **[5]**

5. **5 marks · Paper 1 style · no calculator**

Find an expression for y given that:

$$\frac{dy}{dx} = xe^{x^2-2}$$

and the curve passes through the point $(-\sqrt{2}, 3)$.

6. **6 marks · Paper 1 style · no calculator**

Find the exact area of the region completely enclosed by the curve $y = 4 - x^2$ and the straight line $y = 2 - x$.

7. **4 marks · Paper 1 style · no calculator**

Find the indefinite integral for each of the following expressions:

1. $\int (6x^2 - 4x + 3)\,dx$ [4]
2. $\int \left(\frac{2}{x} + e^x\right) dx$

8. **5 marks · Paper 2 style · calculator**

The gradient function of a curve is given by $\frac{dy}{dx} = 4x^3 - 2x$. The curve passes through the point $(2, 10)$.

1. Find the general expression for y by integrating the gradient function.
2. Use the given boundary condition to find the constant of integration, C, and write down the exact equation of the curve.

9. **7 marks · Paper 2 style · calculator**

A remote-controlled car travels in a straight line. Its acceleration, $a\ \text{m/s}^2$, is given by $a(t) = e^{0.5t}$ for $t \geq 0$. The car has an initial velocity of $2\ \text{m/s}$ and an initial displacement of $0\ \text{m}$ from its starting position.

1. Find an expression for the velocity, $v(t)$, using the given initial condition.
2. Find an expression for the displacement, $s(t)$, using the given initial condition.
3. Calculate the displacement of the car when $t = 4$ seconds.

10. **7 marks · Paper 1 style · no calculator**

The second derivative of a curve is given by $f''(x) = 6x - \frac{4}{x^2}$ for $x > 0$. The curve has a gradient of 2 at the point where $x = 1$. The curve passes through the point $(1, 5)$.

1. Integrate $f''(x)$ to find an expression for the first derivative $f'(x)$.
2. Integrate $f'(x)$ to find the specific equation of the curve $y = f(x)$.

Answers and worked solutions

Question 1

3 marks

Answer: $\dfrac{(2x+1)^6}{12}+C$

$\dfrac{(2x+1)^6}{6}\times\dfrac{1}{2}$ $M1A1$

$\dfrac{(2x+1)^6}{12}+C$ A1

Question 2

3 marks

Answer: $\dfrac{1}{2}\ln(x^2+4)+C$

let $u=x^2+4$, $du=2x\,dx$ M1

$\dfrac{1}{2}\displaystyle\int\frac{1}{u}\,du$ A1

$\dfrac{1}{2}\ln(x^2+4)+C$ A1

Question 3

4 marks

Answer: $-\dfrac{\cos 3x}{3}-\dfrac{e^{-2x}}{2}+C$

$-\dfrac{\cos 3x}{3}$ A1A1

$-\dfrac{e^{-2x}}{2}$ A1

$+C$ A1

Question 4

5 marks

Answer: $f(x)=x^3-\dfrac{2}{x}+5$

$\displaystyle\int\left(3x^2+2x^{-2}\right)dx$ M1

$x^3-2x^{-1}+C$ A1A1

$4=1-2+C$ M1

$f(x)=x^3-\dfrac{2}{x}+5$ A1

Question 5

5 marks

Answer: $y=0.5e^(x^2-2)+2.5$

Integrate by inspection (reverse chain rule): the inner function is $g(x)=x^2-2$ with $g'(x)=2x$, and $x=\frac{1}{2}(2x)$ (M1)

$y=\displaystyle\int xe^{x^2-2}\,dx=\tfrac{1}{2}e^{x^2-2}+C$ A1A1

Substitute $x=-\sqrt{2}$, $y=3$: $3=\frac{1}{2}e^{2-2}+C$ (M1)

$3=0.5+C\implies C=2.5$

$y=\frac{1}{2}e^{x^2-2}+2.5$ A1

Question 6

6 marks

Answer: $9/2$

Equate to find the intersections:

$4-x^2=2-x\implies x^2-x-2=0$ M1

$(x-2)(x+1)=0\implies x=-1$ and $x=2$ A1

The curve is above the line on $-1<x<2$: Area $=\int_{-1}^{2}\big((4-x^2)-(2-x)\big)dx=\int_{-1}^{2}(2+x-x^2)\,dx$ M1

Integrate: $\left[2x+\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{2}$ A1

Substitute the limits:

$\left(4+2-\frac{8}{3}\right)-\left(-2+\frac{1}{2}+\frac{1}{3}\right)=\frac{10}{3}+\frac{7}{6}$ M1

Area $=\frac{27}{6}=\frac{9}{2}\ (=4.5)$ A1

Question 7

4 marks

Answer: (a) $2x^3-2x^2+3x+C$; (b) $2ln(x)+e^x+C$

1. Apply the power rule (add one to the power, divide by the new power):
   $\int(6x^2-4x+3)\,dx=\frac{6x^3}{3}-\frac{4x^2}{2}+3x+C$ (M1)
   $=2x^3-2x^2+3x+C$ A1
2. Use $\int\frac{1}{x}\,dx=\ln|x|$ and $\int e^x\,dx=e^x$:
   $\int\left(\frac{2}{x}+e^x\right)dx=2\ln|x|+e^x+C$ A1A1

Question 8

5 marks

Answer: (a) $y=x^4-x^2+C$; (b) $y=x^4-x^2-2$

1. $y=\int(4x^3-2x)\,dx$. (M1)
   $y=\frac{4x^4}{4}-\frac{2x^2}{2}+C$.
   $y=x^4-x^2+C$. A1A1

2. Substitute $x = 2$ and $y = 10$:
   $10 = (2)^4 - (2)^2 + C$. **(M1)**
   $10 = 16 - 4 + C \implies 10 = 12 + C \implies C = -2$
   .
   Equation of the curve: $y = x^4 - x^2 - 2$. **A1**

Question 9

7 marks

Answer: (a) $v(t) = 2e^(0.5t)$; (b) $s(t) = 4e^(0.5t) - 4$; (c) $25.6\,m$

1. $v(t) = \int a(t)\,dt = \int e^{0.5t}\,dt = \frac{1}{0.5}e^{0.5t} + C = 2e^{0.5t} + C$
   . **(M1)A1**
   $v(0) = 2 \implies 2 = 2e^0 + C \implies 2 = 2 + C \implies C = 0$
   .
   $v(t) = 2e^{0.5t}$. **A1**
2. $s(t) = \int v(t)\,dt = \int 2e^{0.5t}\,dt = \frac{2}{0.5}e^{0.5t} + K = 4e^{0.5t} + K$
   . **(M1)A1**
   $s(0) = 0 \implies 0 = 4e^0 + K \implies K = -4$
   .
   $s(t) = 4e^{0.5t} - 4$. **A1**
3. Substitute $t = 4$:
   $s(4) = 4e^{0.5(4)} - 4 = 4e^2 - 4$.
   $s(4) = 25.556\ldots \implies 25.6\text{ m } (3sf)$. **A1**

Question 10

7 marks

Answer: (a) $f'(x) = 3x^2 + 4/x - 5$; (b) $f(x) = x^3 + 4ln(x) - 5x + 9$

1. Rewrite: $f''(x) = 6x - 4x^{-2}$.
   Integrate: $f'(x) = \frac{6x^2}{2} - \frac{4x^{-1}}{-1} + C$ **(M1)**
   $f'(x) = 3x^2 + \frac{4}{x} + C$ **A1**
   Use $f'(1) = 2$:
   $2 = 3 + 4 + C \implies C = -5$ **(M1)**
   $f'(x) = 3x^2 + \frac{4}{x} - 5$ **A1**
2. Integrate:
   $f(x) = \int (3x^2 + \frac{4}{x} - 5)dx = x^3 + 4\ln x - 5x + K$
   (since $x > 0$) **A1**
   Use $f(1) = 5$:
   $5 = 1 + 4\ln 1 - 5 + K = -4 + K \implies K = 9$
   (M1)
   $f(x) = x^3 + 4\ln x - 5x + 9$ **A1**

Every solution is also on ibmathrevision.com/slaa/integration-exam-practice
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