IB MATHS AA SL · UNIT 3

Geometry and Right Angled Trigonometry: exam practice

11 questions · 57 marks · worked solutions and markschemes

Name _____ Date _____ Score / 57

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/geometry-and-right-angled-trigonometry-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Find the exact length of the line segment joining the 3D coordinates $A(1, 2, 3)$ and $B(4, -2, 5)$.

2. **3 marks · Paper 1 style · no calculator**

A logo comprises a circle of radius r cm and centre O. A sector of the circle with an angle of $124°$ is removed. Find the exact area of the remaining shape in terms of r and π.

3. **3 marks · Paper 1 style · no calculator**

A hiker walks 8 km from A to B on a bearing of $045°$.

a. Write down the bearing of A from B. **[1]**

b. Find the exact distance that B is north of A. **[2]**

4. **5 marks · Paper 1 style · no calculator**

Points A and B have coordinates $(1, -2, 3)$ and $(5, 4, -1)$ respectively in a 3D Cartesian plane. Point O is the origin $(0, 0, 0)$.

1. Find the exact distance between A and B.
2. Find the coordinates of M, the midpoint of $[AB]$.
3. Find the exact distance of OM.

5. **6 marks · Paper 2 style · calculator**

Two points, $P(2, 1, 3)$ and $Q(5, 2, 6)$, are located on an xyz coordinate grid.

1. Find the length of PQ to three significant figures.
2. Find the coordinates of the midpoint of $[PQ]$.
3. Calculate the angle between the line segment $[PQ]$ and the horizontal xy-plane.

6. **4 marks · Paper 2 style · calculator**

A room is a cuboid $ABCDEFGH$ with a horizontal floor $ABCD$, where $AB = 9$ m and $BC = 12$ m. The height of the room is $AE = 10$ m, with E vertically above A and G vertically above C.

(a) Find the length of the floor diagonal AC. **[2]**

(b) Find the angle between the diagonal AG and the floor. **[2]**

7. **5 marks · Paper 1 style · no calculator**

A cuboid $ABCDEFGH$ has $AB = 6$ cm, $BC = 3$ cm and $AE = 2$ cm, where E is vertically above A and G is vertically above C.

(a) Find the length of the diagonal AC. **[2]**

(b) Show that $AG = 7$ cm. **[2]**

(c) Write down the exact value of the sine of the angle between $[AG]$ and the base $ABCD$. **[1]**

8. **9 marks · Paper 2 style · calculator**

A solid right cone has base radius 6 cm and slant height 10 cm.

(a) Find the height of the cone. **[2]**

(b) Find the volume of the cone. **[2]**

(c) Find the total surface area of the cone. **[3]**

(d) Find the angle between the slant edge and the base. **[2]**

9. **6 marks · Paper 2 style · calculator**

A model building is created in the shape of a rectangular-based right pyramid $ABCDE$. The base $ABCD$ is a rectangle with $AB = 4.6\ \text{cm}$ and $BC = 7.2\ \text{cm}$. The apex E is directly above the centre of the base, and each of the four slant edges $[AE]$, $[BE]$, $[CE]$ and $[DE]$ has length $8.3\ \text{cm}$.

1. Calculate the vertical height of the pyramid.
2. Calculate the volume of the model.
3. Find the angle that the triangular face ABE (with base $4.6\ \text{cm}$) makes with the rectangular base $ABCD$.

10. **6 marks · Paper 2 style · calculator**

From a point A on level ground, the angle of elevation of the top T of a vertical tower is $32°$. From a point B, 40 m closer to the foot of the tower and in line with A, the angle of elevation is $51°$.

(a) Find the height of the tower. **[5]**

(b) Find the distance from B to the foot of the tower. **[1]**

11. **7 marks · Paper 2 style · calculator**

A product in the shape of a sphere with radius $3.6\ \text{cm}$ is packed into cuboidal crates measuring $1.7\ \text{m}$ by $0.9\ \text{m}$ with a depth of $22\ \text{cm}$. The spheres are stacked directly on top of and next to each other in a grid formation.

1. Find the maximum number of full spheres that can be packed into a single crate.
2. Calculate the volume of unused, empty space inside a fully packed crate in cm^3.

Answers and worked solutions

Question 1

3 marks

Answer: $\sqrt{29}$

Substitute into the 3D distance formula:

$\sqrt{(4-1)^2+(-2-2)^2+(5-3)^2}$ **M1**

$=\sqrt{9+16+4}$ **A1**

$AB=\sqrt{29}$ **A1**

Question 2

3 marks

Answer: $59\pi r^2/90$

Angle of the remaining sector: $360°-124°=236°$ **M1**

Area $=\frac{236}{360}\times\pi r^2$ **M1**

$=\frac{59}{90}\pi r^2\ \text{cm}^2$ **A1**

Question 3

3 marks

Answer: (a) $225°$; (b) $4\sqrt{2}$ km

a. $225°$ **A1**

b. $8\cos 45°$ **(M1)**

$=4\sqrt{2}$ km **A1**

Question 4

5 marks

Answer: (a) $2\sqrt{17}$; (b) $(3,1,1)$; (c) $\sqrt{11}$

1. Use the distance formula:
   $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.
   Substitute A and B:
   $AB=\sqrt{(5-1)^2+(4-(-2))^2+(-1-3)^2}=\sqrt{4^2+6^2+(-4)^2}$. **(M1)**
   Simplify:
   $AB=\sqrt{16+36+16}=\sqrt{68}=2\sqrt{17}$. **A1**
2. Use the midpoint formula:
   $M=\left(\frac{1+5}{2},\frac{-2+4}{2},\frac{3+(-1)}{2}\right)$. **(M1)**
   $M=(3,1,1)$. **A1**
3. Distance from the origin:
   $OM=\sqrt{3^2+1^2+1^2}=\sqrt{9+1+1}=\sqrt{11}$. **A1**

Question 5

6 marks

Answer: (a) 4.36; (b) $(3.5,1.5,4.5)$; (c) $43.5°$

1. Use the 3D distance formula:
   $PQ=\sqrt{(5-2)^2+(2-1)^2+(6-3)^2}=\sqrt{3^2+1^2+3^2}=\sqrt{19}$. **(M1)**
   $PQ=4.3588\ldots\approx 4.36$ (3 s.f.). **A1**
2. Average the coordinates: midpoint $=\left(\frac{2+5}{2},\frac{1+2}{2},\frac{3+6}{2}\right)=(3.5,1.5,4.5)$. **(M1)A1**
3. Project $[PQ]$ onto the xy–plane. The horizontal distance is $\sqrt{(5-2)^2+(2-1)^2}=\sqrt{10}$ and the vertical rise is $6-3=3$.
   Use tangent in the right-angled triangle:
   $\tan\theta=\frac{3}{\sqrt{10}}$. **M1**
   With the calculator in degree mode:
   $\theta=\arctan\left(\frac{3}{\sqrt{10}}\right)=43.491\ldots\approx 43.5°$. **A1**

Question 6

4 marks

Answer: (a) 15; (b) $33.7°$

(a)

$\sqrt{9^2+12^2}$ **(M1)**

$AC=15\ m$ **A1**

(b)

$\tan\theta=\frac{10}{15}$ **(M1)**

$33.7°$ **A1**

Question 7

5 marks

Answer: (a) $3\sqrt{5}$ cm; (c) $\frac{2}{7}$

(a)

$AC^2=6^2+3^2$ **(M1)**

$AC=\sqrt{45}=3\sqrt{5}$ cm **A1**

(b)

$AG^2=45+2^2$ **M1**

$=49$, so $AG=7$ **A1 AG**

(c)

$\sin\theta = \frac{2}{7}$ A1

Question 8

9 marks

Answer: (a) 8 cm; (b) $302\ \text{cm}^3\ (96\pi)$; (c) $302\ \text{cm}^2\ (96\pi)$; (d) $53.1°$

(a)

$h^2 + 6^2 = 10^2$ (M1)

$h = 8$ cm A1

(b)

$\frac{1}{3}\pi \times 6^2 \times 8$ (M1)

$96\pi = 302\ \text{cm}^3$ A1

(c)

curved surface $\pi \times 6 \times 10 = 60\pi$ (A1)

$60\pi + 36\pi$ (M1)

$96\pi = 302\ \text{cm}^2$ A1

(d)

$\cos\theta = \frac{6}{10}$ (M1)

$53.1°$ A1

Question 9

6 marks

Answer: (a) 7.12 cm; (b) $78.6\ \text{cm}^3$; (c) $63.2°$

1. Base diagonal $= \sqrt{4.6^2 + 7.2^2} = 8.544\ldots$ cm, so the distance from the centre of the base to a corner is $4.272\ldots$ cm.
   Use Pythagoras with the slant edge 8.3 cm:
   $h = \sqrt{8.3^2 - 4.272\ldots^2}$. M1
   $h = \sqrt{68.89 - 18.25} = \sqrt{50.64} = 7.116\ldots \approx 7.12$ cm (3 s.f.). A1
2. Volume
   $= \frac{1}{3} \times \text{base area} \times h = \frac{1}{3}(4.6 \times 7.2)(7.116\ldots)$. (M1)
   Volume $= 78.56\ldots \approx 78.6\ \text{cm}^3$ (3 s.f.). A1
3. Take the cross-section through the apex perpendicular to $[AB]$: a right-angled triangle with vertical side $h = 7.116\ldots$ and horizontal side half of 7.2, i.e. 3.6.
   $\tan\theta = \frac{7.116\ldots}{3.6}$. M1
   $\theta = \arctan(1.9767\ldots) = 63.165\ldots \approx 63.2°$. A1

Question 10

6 marks

Answer: (a) $50.6\ m$; (b) $41.0\ m$

(a)

$h = d\tan 32°$ and $h = (d - 40)\tan 51°$, where d is the distance from A to the foot M A1

$d\tan 32° = (d - 40)\tan 51°$ (M1)

$d = 80.97\ldots$ (A1)

$h = 50.6\ m$ A1

(b)

$41.0\ m$ A1

Question 11

7 marks

Answer: (a) 828; (b) $175000\ \text{cm}^3$

1. Diameter of one sphere $= 2 \times 3.6 = 7.2$ cm. (M1)
   Crate dimensions in cm:
   $170\ \text{cm} \times 90\ \text{cm} \times 22\ \text{cm}$. (M1)
   Spheres along length:
   $\lfloor 170/7.2 \rfloor = \lfloor 23.61 \rfloor = 23$. (A1)
   Spheres along width:
   $\lfloor 90/7.2 \rfloor = \lfloor 12.5 \rfloor = 12$. (A1)
   Spheres along depth:
   $\lfloor 22/7.2 \rfloor = \lfloor 3.06 \rfloor = 3$.
   Total spheres $= 23 \times 12 \times 3 = 828$ spheres. A1
2. Volume of crate
   $= 170 \times 90 \times 22 = 336600\ \text{cm}^3$. (M1)
   Volume of 828 spheres
   $= 828 \times \frac{4}{3}\pi(3.6)^3 = 828 \times 195.43\ldots = 161817.9\ldots\ \text{cm}^3$
   .
   Unused space
   $= 336600 - 161817.9\ldots = 174782\ldots \implies 175000\ \text{cm}^3$ $(3sf)$. A1

Every solution is also on ibmathrevision.com/slaa/geometry-and-right-angled-trigonometry-exam-practice
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