Function fundamentals: exam practice

10 questions · 46 marks · worked solutions and markschemes

Name _____ Date _____ Score / 46

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/function-fundamentals-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Given the functions $f(x) = 2x + 1$ and $g(x) = x^2 - 3$, evaluate exactly $(f \circ g)(2)$.

2. **4 marks · Paper 1 style · no calculator**

A one-to-one function is defined by $f(x) = \frac{1}{2}e^{x-3} + 4$ for $x \in \mathbb{R}$. Find an algebraic expression for the inverse function, $f^{-1}(x)$.

3. **4 marks · Paper 1 style · no calculator**

Let $f(x) = ax + 3$ and $g(x) = 2x - 1$, where a is a constant.

Given that $(f \circ g)(x) = (g \circ f)(x)$ for all x, find the value of a. **[4]**

4. **3 marks · Paper 1 style · no calculator**

A one-to-one function h is defined such that $h(-2) = 5$.

1. Write down the exact value of $h^{-1}(5)$.
2. A new function is defined by $k(x) = 3h(x) + 4$. Find the exact value of $k(-2)$.

5. **3 marks · Paper 1 style · no calculator**

Let $f(x) = 3x - 7$.

a. Find $f(4)$. **[1]**

b. Find the value of x for which $f(x) = 11$. **[2]**

6. **4 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{1}{\sqrt{6 - 2x}}$.

(a) Find the largest possible domain of f. **[2]**

(b) Find $f(-5)$. **[2]**

7. **5 marks · Paper 1 style · no calculator**

Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$.

(a) Find $(g \circ f)(x)$, giving your answer in the form $ax^2 + bx + c$. **[2]**

(b) Solve $(f \circ g)(x) = 15$. **[3]**

8. **6 marks · Paper 1 style · no calculator**

Let $h(x) = \sqrt{x - 4}$, for $x \geq 4$.

(a) Write down the range of h. **[1]**

(b) Find $h^{-1}(x)$ and state its domain. **[3]**

(c) Find the value of x for which $h(x) = 5$. **[2]**

9. 6 marks · Paper 1 style · no calculator

Let $f(x) = e^{2x}$ and $g(x) = \ln(x+1)$, for $x > -1$.

(a) Show that $(f \circ g)(x) = (x+1)^2$. **[2]**

(b) Solve $(f \circ g)(x) = 9$. **[2]**

(c) Find $f^{-1}(x)$. **[2]**

10. 8 marks · Paper 2 style · calculator

Let $f(x) = x^2 - 4x + 1$, for $x \geq 2$.

(a) Find the range of f. **[2]**

(b) Find $f^{-1}(x)$. **[3]**

(c) Solve $f(x) = f^{-1}(x)$. **[3]**

Answers and worked solutions

Question 1

3 marks

Answer: 3

Evaluate the inner function first: $g(2) = 2^2 - 3$. **M1**

$g(2) = 1$ **A1**

Evaluate the outer function: $f(1) = 2(1) + 1 = 3$. **A1**

Question 2

4 marks

Answer: $f^-1(x) = \ln(2x - 8) + 3$

Swap x and y: $x = \frac{1}{2}e^{y-3} + 4$. **M1**

Isolate the exponential: $2(x - 4) = e^{y-3}$. **M1**

Take natural logarithms: $\ln(2x - 8) = y - 3$. **M1**

$f^{-1}(x) = \ln(2x - 8) + 3$ **A1**

Question 3

4 marks

Answer: -2

$(f \circ g)(x) = a(2x - 1) + 3 = 2ax - a + 3$ **A1**

$(g \circ f)(x) = 2(ax + 3) - 1 = 2ax + 5$ **A1**

$-a + 3 = 5$ **M1**

$a = -2$ **A1**

Question 4

3 marks

Answer: (a) -2; (b) 19

1. Since $h(-2) = 5$, the inverse gives $h^{-1}(5) = -2$. **A1**
2. Substitute $h(-2) = 5$:
   $k(-2) = 3h(-2) + 4 = 3(5) + 4$. **M1**
   $k(-2) = 19$ **A1**

Question 5

3 marks

Answer: (a) 5; (b) $x = 6$

a. $f(4) = 12 - 7 = 5$ **A1**

b. $3x - 7 = 11$ **(M1)**
   $x = 6$ **A1**

Question 6

4 marks

Answer: (a) $x < 3$; (b) $\frac{1}{4}$

(a)

$6 - 2x > 0$ **(M1)**

$x < 3$ **A1**

(b)

$\frac{1}{\sqrt{16}}$ **(M1)**

$\frac{1}{4}$ **A1**

Question 7

5 marks

Answer: (a) $4x^2 - 12x + 10$; (b) $x = \pm 2\sqrt{2}$

(a)

$(2x - 3)^2 + 1$ **(M1)**

$4x^2 - 12x + 10$ **A1**

(b)

$2(x^2 + 1) - 3 = 15$ **M1**

$x^2 = 8$ **A1**

$x = \pm 2\sqrt{2}$ **A1**

Question 8

6 marks

Answer: (a) $h(x) \geq 0$; (b) $h^{-1}(x) = x^2 + 4$, $x \geq 0$; (c) 29

(a)

$h(x) \geq 0$ **A1**

(b)

$x = \sqrt{y - 4}$ **M1**

$h^{-1}(x) = x^2 + 4$ **A1**

domain $x \geq 0$ **A1**

(c)

$x - 4 = 25$ **(M1)**

$x = 29$ **A1**

Question 9

6 marks

Answer: (b) 2; (c) $f^{-1}(x) = \frac{1}{2}\ln x$

(a)

$e^{2\ln(x+1)}$ **M1**

$= e^{\ln(x+1)^2} = (x+1)^2$ **A1 AG**

(b)

$x + 1 = 3$ (as $x + 1 > 0$) **(M1)**

$x = 2$ **A1**

(c)

$x = e^{2y}$ **M1**

$f^{-1}(x) = \frac{1}{2}\ln x$ **A1**

Question 10

8 marks

Answer: (a) $f(x) \geq -3$; (b) $f^{-1}(x) = 2 + \sqrt{x+3}$; (c) $x = \frac{5+\sqrt{21}}{2}\ (= 4.79)$

(a)

minimum at $x = 2$, $f(2) = -3$ **(M1)**

$f(x) \geq -3$ **A1**

(b)

$x = (y-2)^2 - 3$ **M1**

$y - 2 = \pm\sqrt{x+3}$ **A1**

$f^{-1}(x) = 2 + \sqrt{x+3}$ (positive root since $y \geq 2$) **A1**

(c)

solutions lie on $y = x$: $x^2 - 4x + 1 = x$ **(M1)**

$x^2 - 5x + 1 = 0$ **(A1)**

$x = \frac{5+\sqrt{21}}{2}\ (4.79)$ **A1**

Every solution is also on ibmathrevision.com/slaa/function-fundamentals-exam-practice
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