IB MATHS AA SL · UNIT 1

Exponentials and Logarithms: exam practice

10 questions · 53 marks · worked solutions and markschemes

Name _____ Date _____ Score / 53

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/exponentials-and-logarithms-exam-practice

1. **4 marks · Paper 1 style · no calculator**

Solve $2^{x+1} = 3^x$, giving your answer in the form $x = \dfrac{\ln a}{\ln b}$, where $a, b \in \mathbb{Q}$. **[4]**

2. **6 marks · Paper 1 style · no calculator**

Solve the equation $\ln(x+2) + \ln(x-1) = \ln(4x)$ for $x > 1$.

3. **6 marks · Paper 2 style · calculator**

Solve the equation $4^x - 3 \times 2^{x+2} = 64$.

4. **4 marks · Paper 1 style · no calculator**

Simplify fully, giving each answer as a single power of 2.

(a) $\dfrac{8^3 \times 4^{-1}}{16}$ **[2]**

(b) $\sqrt{32} \times \left(\frac{1}{2}\right)^5$ **[2]**

5. **5 marks · Paper 1 style · no calculator**

Find the exact value of each of the following:

1. $\log_2 16$
2. $\log 25 + \log 4$
3. $\log_5 500 - \log_5 4$

6. **6 marks · Paper 1 style · no calculator**

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y:

1. $\ln 5$
2. $\ln 45$
3. $\ln 135$

7. **6 marks · Paper 2 style · calculator**

Let $f(x) = \ln(x + 2)$ for $x > -2$.

1. Find the exact coordinates of the x-intercept and the y-intercept.
2. State the equation of the vertical asymptote to the graph of f.
3. The graph of $y = f(x)$ intersects with its inverse, $y = f^{-1}(x)$, twice. Using your GDC, find the coordinates of these two points of intersection.

8. **6 marks · Paper 1 style · no calculator**

(a) Write $\log_2 48 - \log_2 3$ as an integer. **[2]**

(b) Given that $\log_3 a = p$, find an expression for $\log_9(27a^2)$ in terms of p. **[4]**

9. **6 marks · Paper 2 style · calculator**

The number of bacteria in a dish t hours after the start of an experiment is $N = 500e^{0.35t}$.

(a) Write down the number of bacteria at the start. **[1]**

(b) Find the number of bacteria after 4 hours. **[2]**

(c) Find the time, in hours, for the number of bacteria to reach $10\,000$. Give your answer to 3 significant figures. **[3]**

10. **4 marks · Paper 1 style · no calculator**

Let $x = 2^p$ and $y = 2^q$, where $p, q \in \mathbb{R}$. Each answer below is of the form 2^m. Write down m in terms of p and q for

a. $8xy^2$; **[2]**

b. $\dfrac{\sqrt{x}}{y^3}$. **[2]**

Answers and worked solutions

Question 1

4 marks

Answer: $x = \dfrac{\ln 2}{\ln 1.5}$

$(x+1)\ln 2 = x\ln 3$ M1

$x\ln 3 - x\ln 2 = \ln 2$ (A1)

$x(\ln 3 - \ln 2) = \ln 2$, so $x = \dfrac{\ln 2}{\ln \frac{3}{2}}$ A1A1

Question 2

6 marks

Answer: $x = (3+\sqrt{17})/2$

Combine the logarithms on the LHS:

$\ln\left((x+2)(x-1)\right) = \ln(4x)$ M1

Equate the arguments: $(x+2)(x-1) = 4x$ M1

Expand: $x^2 + x - 2 = 4x$

Form the quadratic: $x^2 - 3x - 2 = 0$ A1

Use the quadratic formula:

$x = \frac{3\pm\sqrt{9+8}}{2} = \frac{3\pm\sqrt{17}}{2}$ M1 A1

Reject $\frac{3-\sqrt{17}}{2} < 0$ since $x > 1$ is required, so

$x = \frac{3+\sqrt{17}}{2}$ A1

Question 3

6 marks

Answer: $x = 4$

Rewrite using base 2:

$(2^x)^2 - 3\times(2^2\times 2^x) = 64$. (M1)

$(2^x)^2 - 12(2^x) - 64 = 0$. (A1)

Let $u = 2^x$: $u^2 - 12u - 64 = 0$. (M1)

CG50: `MENU A (Equation) -> Polynomial`, `a=1, b=-12, c=-64` (or factorise $(u-16)(u+4)=0$).

$u = 16$ or $u = -4$. A1

$2^x = -4$ has no real solution since $2^x > 0$. R1

$2^x = 16 \implies x = 4$. A1

Question 4

4 marks

Answer: (a) 2^3; (b) $2^{-5/2}$

(a)

$\dfrac{2^9\times 2^{-2}}{2^4}$ M1

2^3 A1

(b)

$2^{5/2}\times 2^{-5}$ M1

$2^{-5/2}$ A1

Question 5

5 marks

Answer: (a) 4; (b) 2; (c) 3

1. Since $2^4 = 16$, $\log_2 16 = 4$. A1
2. Use the product law:
   $\log 25 + \log 4 = \log(25\times 4)$. (M1)
   $= \log 100 = 2$. A1
3. Use the quotient law:
   $\log_5 500 - \log_5 4 = \log_5\left(\frac{500}{4}\right) = \log_5 125$. (M1)
   Since $5^3 = 125$, the value is 3. A1

Question 6

6 marks

Answer: (a) $x - y$; (b) $x + y$; (c) $x + 2y$

1. $\ln 5 = \ln\left(\frac{15}{3}\right) = \ln 15 - \ln 3$. (M1)
   $= x - y$. A1
2. $\ln 45 = \ln(15\times 3) = \ln 15 + \ln 3$. (M1)
   $= x + y$. A1
3. $\ln 135 = \ln(15\times 9) = \ln(15\times 3^2) = \ln 15 + 2\ln 3$. (M1)
   $= x + 2y$. A1

Question 7

6 marks

Answer: (a) $(-1,0),(0,ln2)$; (b) $x = -2$; (c) $(-1.84,-1.84),(1.15,1.15)$

1. x—intercept: set $y = 0$:
   $\ln(x+2) = 0 \implies x+2 = e^0 = 1 \implies x = -1$, giving $(-1,0)$. A1
   y—intercept: set $x = 0$: $y = \ln 2$, giving $(0,\ln 2)$. A1
2. The vertical asymptote is where the argument of $\ln$ is zero: $x + 2 = 0$, so $x = -2$. A1

3. A function and its inverse meet on the line $y = x$, so solve $\ln(x+2) = x$. **(M1)**
   CG50: `MENU 5 (Graph)`, `Y1 = ln(x+2)`, `Y2 = x`, `DRAW`, then `G-Solv -> F5 (ISCT)`.
   Coordinates: $(-1.84, -1.84)$ and $(1.15, 1.15)$. **A1 A1**

Question 8

6 marks

Answer: (a) 4; (b) $\dfrac{3+2p}{2}$

(a)

$\log_2 \dfrac{48}{3} = \log_2 16$ **M1**

$= 4$ **A1**

(b)

$\log_9(27a^2) = \dfrac{\log_3(27a^2)}{\log_3 9}$ **M1**

$\log_3 9 = 2$ **(A1)**

$\log_3(27a^2) = 3 + 2p$ **A1**

$\dfrac{3+2p}{2}$ **A1**

Question 9

6 marks

Answer: (a) 500; (b) 2030; (c) $t = 8.56$ hours

(a)

500 **A1**

(b)

$500e^{0.35\times 4}$ **(M1)**

2030 (2027.6…) **A1**

(c)

$500e^{0.35t} = 10\,000$ **(M1)**

$0.35t = \ln 20$ **(A1)**

$t = 8.56$ hours (8.5593…) **A1**

Question 10

4 marks

Answer: (a) $3 + p + 2q$; (b) $p/2 - 3q$

a. $8 = 2^3$, $y^2 = 2^{2q}$ **(M1)**
   $8xy^2 = 2^{3+p+2q}$, so $m = 3 + p + 2q$ **A1**

b. $\sqrt{x} = 2^{\frac{p}{2}}$, $y^3 = 2^{3q}$ **(M1)**
   $m = \dfrac{p}{2} - 3q$ **A1**

Every solution is also on ibmathrevision.com/slaa/exponentials-and-logarithms-exam-practice
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