IB MATHS AA SL · UNIT 2

Exponential and Logarithmic Functions: exam practice

10 questions · 49 marks · worked solutions and markschemes

Name _____ Date _____ Score / 49

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/exponential-and-logarithmic-functions-exam-practice

1. **5 marks · Paper 1 style · no calculator**

Solve the equation $\log_3(x+1)+\log_3(x-1)=1$. **[5]**

2. **2 marks · Paper 1 style · no calculator**

The graph of $y=\log_3 x$ passes through the points $(k,2)$ and $\left(\frac{1}{3},m\right)$. Find k and m.

3. **4 marks · Paper 1 style · no calculator**

Solve the following equations exactly:

1. $e^{3x-1}=20$
2. $\ln(2x)=5$

4. **5 marks · Paper 1 style · no calculator**

Given that $\log_a 3 = x$ and $\log_a 4 = y$. Write down expressions for the following in terms of x and y:

1. $\log_a 12$
2. $\log_a 0.75$
3. $\log_a 36$

5. **5 marks · Paper 2 style · calculator**

The radioactive mass, M in grams, of a substance after t days is given by $M(t) = 40e^{-kt}$, where k is a positive constant.

1. Given that the mass halves after exactly 12 days, use your GDC's equation solver to find the value of k to four significant figures.
2. Calculate the mass remaining after 30 days.

6. **5 marks · Paper 2 style · calculator**

Let $g(x) = \ln(x-2) + 1$, for $x > 2$.

(a) Write down the equation of the vertical asymptote of the graph of g. **[1]**

(b) Find the x-intercept of the graph of g. **[2]**

(c) Find $g^{-1}(x)$. **[2]**

7. **5 marks · Paper 1 style · no calculator**

The graph of $y = a \times b^x$, where $b > 0$, passes through the points $(0, 3)$ and $(2, 48)$.

(a) Find the value of a and the value of b. **[3]**

(b) Find the value of x for which $y = 768$. **[2]**

8. **6 marks · Paper 1 style · no calculator**

Let $f(x) = 3 + 2e^{-x}$, for $x \in \mathbb{R}$.

(a) Write down the y-intercept of the graph of f. **[1]**

(b) Write down the equation of the horizontal asymptote. **[1]**

(c) Find $f^{-1}(x)$, and state its domain. **[4]**

9. **7 marks · Paper 2 style · calculator**

A cup of coffee cools so that its temperature, $T\,°\text{C}$, t minutes after it is poured is $T = 20 + 75e^{-0.04t}$.

(a) Write down the temperature when the coffee is poured. **[1]**

(b) Find the temperature after 10 minutes. **[2]**

(c) Find how long it takes for the temperature to fall to $40\,°\text{C}$. **[3]**

(d) State what happens to the temperature for large values of t. **[1]**

10. **5 marks · Paper 1 style · no calculator**

(a) Show that $\ln(x^2 - 9) - \ln(x - 3) = \ln(x + 3)$, for $x > 3$. **[2]**

(b) Hence solve $\ln(x^2 - 9) - \ln(x - 3) = 2$, giving your answer in exact form. **[3]**

Answers and worked solutions

Question 1

5 marks

Answer: 2

$\log_3((x+1)(x-1)) = 1$ M1

$x^2 - 1 = 3$ A1

$x = \pm 2$ (A1)

$x = -2$ rejected since $x - 1 < 0$ R1

$x = 2$ A1

Question 2

2 marks

Answer: $k = 9$; $m = -1$

$\log_3 k = 2$, so $k = 9$ A1

$m = \log_3 \frac{1}{3} = -1$ A1

Question 3

4 marks

Answer: (a) $x = (ln20 + 1)/3$; (b) $x = e^5/2$

1. Take the natural logarithm of both sides: $\ln(e^{3x-1}) = \ln 20$. (M1)
   $3x - 1 = \ln 20 \implies 3x = \ln 20 + 1 \implies x = \frac{\ln 20 + 1}{3}$. A1
2. Convert from logarithmic to exponential form: $e^5 = 2x$. (M1)
   $x = \frac{e^5}{2}$. A1

Question 4

5 marks

Answer: (a) $x + y$; (b) $x - y$; (c) $2x + y$

1. $\log_a 12 = \log_a(3 \times 4) = \log_a 3 + \log_a 4$.
   $= x + y$. A1
2. $\log_a 0.75 = \log_a\left(\frac{3}{4}\right) = \log_a 3 - \log_a 4$. (M1)
   $= x - y$. A1
3. $\log_a 36 = \log_a(9 \times 4) = \log_a(3^2) + \log_a 4 = 2\log_a 3 + \log_a 4$. (M1)
   $= 2x + y$. A1

Question 5

5 marks

Answer: (a) $k = 0.05776$; (b) $7.07 grams$

1. Set $M(12) = 20$ (half of the initial $40g$).
   $40e^{-12k} = 20 \implies e^{-12k} = 0.5$. (M1)
   CG50 Equation Solver: `e^(-12x) = 0.5`. (M1)
   $k = 0.057762... \implies k = 0.05776$ $(4sf)$. A1
2. $M(30) = 40e^{-0.05776 \times 30}$. (M1)
   CG50: `Run-Matrix`, evaluate `40 * e^(-0.05776 * 30)`.
   $M = 7.071... \implies 7.07$ grams. A1

Question 6

5 marks

Answer: (a) $x = 2$; (b) $x = 2 + e^{-1}$ $(= 2.37)$; (c) $g^{-1}(x) = e^{x-1} + 2$

(a)

$x = 2$ A1

(b)

$\ln(x - 2) = -1$ (M1)

$x = 2 + e^{-1}$ (2.37) A1

(c)

$x - 1 = \ln(y - 2)$ M1

$g^{-1}(x) = e^{x-1} + 2$ A1

Question 7

5 marks

Answer: (a) $a = 3,\ b = 4$; (b) 4

(a)

$a = 3$ A1

$3b^2 = 48$ M1

$b = 4$ A1

(b)

$4^x = 256$ (M1)

$x = 4$ A1

Question 8

6 marks

Answer: (a) $(0, 5)$; (b) $y = 3$; (c) $f^{-1}(x) = \ln\frac{2}{x-3}$, $x > 3$

(a)

$(0, 5)$ **A1**

(b)

$y = 3$ **A1**

(c)

$x = 3 + 2e^{-y}$ **M1**

$e^{-y} = \dfrac{x-3}{2}$ **A1**

$f^{-1}(x) = -\ln\left(\dfrac{x-3}{2}\right) \left(= \ln \dfrac{2}{x-3}\right)$ **A1**

domain $x > 3$ **A1**

Question 9

7 marks

Answer: (a) $95\,°\text{C}$; (b) $70.3\,°\text{C}$; (c) 33.0 minutes; (d) it approaches $20\,°\text{C}$

(a)

$95\,°\text{C}$ **A1**

(b)

$20 + 75e^{-0.4}$ **(M1)**

$70.3\,°\text{C}$ **A1**

(c)

$75e^{-0.04t} = 20$ **(M1)**

$t = \dfrac{\ln(20/75)}{-0.04}$ **(A1)**

33.0 minutes **A1**

(d)

it approaches $20\,°\text{C}$ (room temperature) **A1**

Question 10

5 marks

Answer: (b) $x = e^2 - 3$

(a)

$\ln \dfrac{x^2-9}{x-3}$ **M1**

$= \ln \dfrac{(x-3)(x+3)}{x-3} = \ln(x+3)$ **A1 AG**

(b)

$\ln(x+3) = 2$ **(M1)**

$x + 3 = e^2$ **A1**

$x = e^2 - 3$ **A1**

Every solution is also on ibmathrevision.com/slaa/exponential-and-logarithmic-functions-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.