Discrete Random Variables: exam practice

10 questions · 50 marks · worked solutions and markschemes

Name _____ Date _____ Score / 50

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/discrete-random-variables-exam-practice

1. **4 marks · Paper 2 style · calculator**

A basketball player scores from a free throw with probability 0.2, independently each time.

Find the least number of free throws she must take so that the probability of scoring at least once is greater than 0.95. **[4]**

2. **4 marks · Paper 2 style · calculator**

A fair coin is tossed 20 times and the number of times it lands heads up is recorded as the random variable X.

1. Find the expected number of times that the coin will land heads up.
2. Use your graphic display calculator to find the probability that the coin lands heads up exactly 15 times.

3. **4 marks · Paper 2 style · calculator**

The random variable X is binomially distributed such that $X \sim B(40, 0.15)$. Find the values of:

1. $P(3 \leq X < 14)$
2. $\text{Var}(X)$

4. **5 marks · Paper 1 style · no calculator**

A discrete random variable X can take the values $1, 2, 3,$ and 4. The probabilities are given as follows: $P(X = 1) = 0.2$, $P(X = 2) = 0.3$, $P(X = 3) = a$, and $P(X = 4) = b$. The expected value of the distribution is $E(X) = 2.6$.

Find the exact values of a and b.

5. **5 marks · Paper 1 style · no calculator**

The probability distribution of a discrete random variable X is given by the formula:

$$P(X = x) = \frac{x}{k} \quad \text{for } x \in \{1, 2, 3, 4\}$$

where k is a positive constant.

1. Find the exact value of k.
2. Find the expected value, $E(X)$.

6. **6 marks · Paper 2 style · calculator**

In a fairground game a player loses €2 with probability 0.5, wins €1 with probability 0.3 and wins €10 with probability 0.2. Let X be the player's gain, in euros, in one game.

a. Find $\mathrm{E}(X)$. **[2]**

b. Find $\mathrm{Var}(X)$. **[3]**

c. Find the standard deviation of X. **[1]**

7. **6 marks · Paper 1 style · no calculator**

The random variable X has probability distribution $P(X = x) = k(x^2 + 1)$, for $x \in \{0, 1, 2, 3\}$.

(a) Find the value of k. **[2]**

(b) Find $\mathrm{E}(X)$. **[2]**

(c) Find $P(X \geq 2)$. **[2]**

8. 6 marks · Paper 2 style · calculator

In a large school, 30% of students walk to school. A random sample of 12 students is chosen, and X is the number who walk to school.

(a) Write down the distribution of X. **[1]**

(b) Find $P(X = 4)$. **[2]**

(c) Find the probability that at least 4 of the students walk to school. **[2]**

(d) Find the expected number of students who walk. **[1]**

9. 5 marks · Paper 2 style · calculator

A company manufactures computer chips. It is known that 12% of the chips produced are defective. A quality control inspector takes a random sample of 50 chips. Let the random variable D represent the number of defective chips in the sample.

1. Find the probability that the inspector finds fewer than 8 defective chips.
2. Given that the inspector finds fewer than 8 defective chips, find the conditional probability that they found exactly 5 defective chips.

10. **5 marks · Paper 1 style · no calculator**

In a game, a player rolls two fair six-sided dice. The player wins €30 if both dice show a six, wins €6 if exactly one die shows a six, and wins nothing otherwise.

(a) Find the probability that exactly one die shows a six. **[2]**

(b) Find the expected amount won in one game. **[2]**

(c) The player pays €c to play. Find the value of c that makes the game fair. **[1]**

Answers and worked solutions

Question 1

4 marks

Answer: 14

$P(X \geq 1) = 1 - 0.8^n$ (M1)

$1 - 0.8^n > 0.95$ M1

$n > 13.4\ldots$ (A1)

$n = 14$ A1

Question 2

4 marks

Answer: (a) 10; (b) 0.0148

Let X be the number of heads. $X \sim B(20, 0.5)$. (M1)

1. Expected value
   $E(X) = np = 20 \times 0.5 = 10$. A1
2. CG50: `STAT -> DIST -> BINOMIAL -> Bpd`.
   Set $x = 15$, $\text{Numtrial} = 20$, $p = 0.5$. (M1)
   $P(X = 15) = 0.014785\ldots \implies 0.0148$ $(3sf)$. A1

Question 3

4 marks

Answer: (a) 0.950; (b) 5.1

1. $P(3 \leq X < 14)$ is equivalent to $P(3 \leq X \leq 13)$ since X is discrete. (M1)
   CG50: `STAT -> DIST -> BINOMIAL -> Bcd`.
   Set $\text{Lower} = 3$, $\text{Upper} = 13$, $\text{Numtrial} = 40$, $p = 0.15$. (M1)
   $P = 0.9500\ldots \implies 0.950$ $(3sf)$. A1
2. $\text{Var}(X) = np(1-p) = 40(0.15)(0.85)$.
   $\text{Var}(X) = 6 \times 0.85 = 5.1$. A1

Question 4

5 marks

Answer: $a = 0.2$; $b = 0.3$

Sum of the probabilities equals 1:

$0.2 + 0.3 + a + b = 1 \implies a + b = 0.5$. M1

Set up the expected value equation:

$1(0.2) + 2(0.3) + 3a + 4b = 2.6$. M1

Simplify:

$0.8 + 3a + 4b = 2.6 \implies 3a + 4b = 1.8$. A1

Solve simultaneously, e.g.

$3(0.5 - b) + 4b = 1.8 \implies 1.5 + b = 1.8$. M1

$b = 0.3$ and $a = 0.2$. A1

Question 5

5 marks

Answer: (a) $k = 10$; (b) 3

1. Sum of probabilities $= 1$.
   $\frac{1}{k} + \frac{2}{k} + \frac{3}{k} + \frac{4}{k} = 1$. (M1)
   $\frac{10}{k} = 1 \implies k = 10$. A1
2. $E(X) = \sum xP(X = x) = 1\left(\frac{1}{10}\right) + 2\left(\frac{2}{10}\right) + 3\left(\frac{3}{10}\right) + 4\left(\frac{4}{10}\right)$. (M1)
   $E(X) = \frac{1+4+9+16}{10}$. (A1)
   $E(X) = \frac{30}{10} = 3$. A1

Question 6

6 marks

Answer: (a) 1.3; (b) 20.61; (c) 4.54

a. $-2 \times 0.5 + 1 \times 0.3 + 10 \times 0.2$ (M1)
   1.3 A1

b. $\mathrm{E}\left(X^2\right) = 4 \times 0.5 + 1 \times 0.3 + 100 \times 0.2 = 22.3$
   $M1$A1
   $22.3 - 1.3^2 = 20.61$ A1

c. $\sqrt{20.61} = 4.54$ A1

Question 7

6 marks

Answer: (a) $\frac{1}{18}$; (b) $\frac{7}{3}$; (c) $\frac{5}{6}$

(a)

$k(1 + 2 + 5 + 10) = 1$ M1

$k = \frac{1}{18}$ A1

(b)

$\frac{0 \times 1 + 1 \times 2 + 2 \times 5 + 3 \times 10}{18}$ (M1)

$\frac{42}{18} = \frac{7}{3}$ A1

(c)

$\frac{5 + 10}{18}$ (M1)

$\frac{5}{6}$ **A1**

Question 8

6 marks

Answer: (b) 0.231; (c) 0.507; (d) 3.6

(a)

$X \sim B(12,\ 0.3)$ **A1**

(b)

binomial pdf **(M1)**

0.231 **A1**

(c)

$1 - P(X \leq 3)$ **(M1)**

0.507 **A1**

(d)

$12 \times 0.3 = 3.6$ **A1**

Question 9

5 marks

Answer: (a) 0.753; (b) 0.222

$D \sim B(50, 0.12)$.

1. "Fewer than 8" means $P(D \leq 7)$. **(M1)**
   CG50: Bcd with
   $\mathbf{Lower} = 0, \mathbf{Upper} = 7, \mathbf{Numtrial} = 50, p = 0.12$
   .
   $P(D \leq 7) = 0.75325\ldots \implies 0.753$ (3 s.f.). **A1**
2. Conditional probability:
   $P(D = 5 \mid D \leq 7) = \frac{P(D=5 \cap D \leq 7)}{P(D \leq 7)}$. **(M1)**
   Since the event $D = 5$ is contained in $D \leq 7$, the intersection is just $D = 5$:
   $P(D = 5 \mid D \leq 7) = \frac{P(D=5)}{P(D \leq 7)}$. **(M1)**
   CG50: Bpd with $x = 5$:
   $P(D = 5) = 0.16738\ldots$
   Probability
   $= \frac{0.16738\ldots}{0.75325\ldots} = 0.22221\ldots \implies 0.222$ (3 s.f.). **A1**

Question 10

5 marks

Answer: (a) $\frac{5}{18}$; (b) €2.50; (c) $c = 2.50$

(a)

$2 \times \frac{1}{6} \times \frac{5}{6}$ **(M1)**

$\frac{10}{36} = \frac{5}{18}$ **A1**

(b)

$30 \times \frac{1}{36} + 6 \times \frac{10}{36}$ **(M1)**

€2.50 **A1**

(c)

$c = 2.50$ **A1**

Every solution is also on ibmathrevision.com/slaa/discrete-random-variables-exam-practice
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