Definite Integrals: exam practice

10 questions · 42 marks · worked solutions and markschemes

Name _____ Date _____ Score / 42

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/definite-integrals-exam-practice

1. **4 marks · Paper 1 style · no calculator**

Find the exact area of the region bounded by the curve $y = e^{0.5x}$, the x-axis, the y-axis, and the vertical line $x = 2$.

2. **4 marks · Paper 1 style · no calculator**

Find the exact value of the positive constant k such that:

$$\int_0^k 3x^2 \, dx = 64$$

3. **3 marks · Paper 2 style · calculator**

Let $f(x) = xe^{-x}$.

Find the area of the region enclosed by the graph of f, the x-axis and the line $x = 3$. **[3]**

4. **4 marks · Paper 1 style · no calculator**

Evaluate exactly the definite integral $\int_0^{\ln 2} 4e^{2x}\,dx$.

5. **4 marks · Paper 1 style · no calculator**

Evaluate exactly the definite integral $\int_0^{\frac{\pi}{4}} \cos(2x)\,dx$.

6. **4 marks · Paper 1 style · no calculator**

Find the area of the region between the graph of $y = 4\sin\left(\frac{x}{2}\right)$ and the x-axis, for $0 \leq x \leq 2\pi$. **[4]**

7. **5 marks · Paper 1 style · no calculator**

Find the exact value of $\int_1^4 \left(3\sqrt{x} - \frac{2}{x^2}\right) dx$. **[5]**

8. **4 marks · Paper 2 style · calculator**

Consider the function $f(x) = \frac{1}{2}x^2 + 2$.

1. Using your graphic display calculator, calculate the exact value of the definite integral $\int_0^4 \left(\frac{1}{2}x^2 + 2\right) dx$.
2. A student estimates the area under the curve between $x = 0$ and $x = 4$ using a basic geometric method and gets an answer of 19. Calculate the percentage error of their approximation.

9. **4 marks · Paper 1 style · no calculator**

It is given that $\int_1^3 f(x)\,dx = 7$.

(a) Write down $\int_3^1 f(x)\,dx$. **[1]**

(b) Find $\int_1^3 (2f(x) + x)dx$. **[3]**

10. **6 marks · Paper 1 style · no calculator**

The line $y = 3x$ cuts the parabola $y = x^2$ at $O(0, 0)$ and at a second point B.

(a) Write down the coordinates of B. **[2]**

(b) Calculate the exact area between the line and the parabola. **[4]**

Answers and worked solutions

Question 1

4 marks

Answer: $2e - 2$

Set up the definite integral: $\int_0^2 e^{0.5x}\,dx$ M1

Integrate: $\left[\frac{1}{0.5}e^{0.5x}\right]_0^2 = \left[2e^{0.5x}\right]_0^2$ A1

Substitute the limits: $2e^{0.5(2)} - 2e^0$ (M1)

$= 2e - 2$ A1

Question 2

4 marks

Answer: $k = 4$

Integrate analytically: $\int 3x^2\,dx = x^3$. (M1)

Evaluate from 0 to k: $[x^3]_0^k = k^3 - 0^3 = k^3$. (M1)A1

Equate to 64: $k^3 = 64 \implies k = \sqrt[3]{64} = 4$. A1

Question 3

3 marks

Answer: 0.801

$\int_0^3 xe^{-x}\,dx$ (M1)(A1)

0.801 A1

Question 4

4 marks

Answer: 6

Integrate: $\int 4e^{2x}\,dx = \left[\frac{4}{2}e^{2x}\right]_0^{\ln 2} = \left[2e^{2x}\right]_0^{\ln 2}$ M1 A1

Substitute the limits:

$2e^{2\ln 2} - 2e^0 = 2e^{\ln 4} - 2 = 2(4) - 2$ M1

$= 6$ A1

Question 5

4 marks

Answer: $1/2$

Integrate: $\left[\frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{4}}$ M1 A1

Substitute the limits:

$\frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin 0 = \frac{1}{2}(1) - 0$ M1

$= \frac{1}{2}$ A1

Question 6

4 marks

Answer: 16

$\int_0^{2\pi} 4\sin\frac{x}{2}\,dx$ M1

$\left[-8\cos\frac{x}{2}\right]_0^{2\pi}$ A1

$8 - (-8)$ (M1)

16 A1

Question 7

5 marks

Answer: $\frac{25}{2}$

$\left[2x^{3/2} + \frac{2}{x}\right]_1^4$ A1A1

$\left(16 + \frac{1}{2}\right) - (2 + 2)$ M1

$= \frac{25}{2}$ A2

Question 8

4 marks

Answer: (a) $56/3$; (b) 1.79

1. **CG50:** Run-Matrix $\rightarrow$ MATH $\rightarrow \int$dx.
   Enter $\int_0^4 (0.5x^2 + 2)\,dx$. (M1)
   Exact Area $= 18.666... = \frac{56}{3}$. A1
2. $\text{Percentage Error} = \left|\frac{\text{Approximate} - \text{Exact}}{\text{Exact}}\right| \times 100\%$. (M1)
   $= \left|\frac{19 - 18.666...}{18.666...}\right| \times 100\%$.
   Error $= 1.7857... \approx 1.79\%$. A1

Question 9

4 marks

Answer: (a) -7; (b) 18

(a)

-7 A1

(b)

$2 \times 7 + \int_1^3 x\,dx$ M1

$\left[\frac{x^2}{2}\right]_1^3 = 4$ A1

18 A1

Question 10

6 marks

Answer: (a) $(3, 9)$; (b) $\frac{9}{2}$

(a)

$x^2 = 3x$, so $x = 3$ **(M1)**

$B(3, 9)$ **A1**

(b)

$\int_0^3 (3x - x^2)\,dx$ $M1$**A1**

$\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{2} - 9$ **A1**

$\frac{9}{2}$ **A1**

Every solution is also on ibmathrevision.com/slaa/definite-integrals-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.