IB MATHS AA SL · UNIT 4

# Bivariate Data and Correlation: exam practice

10 questions · 49 marks · worked solutions and markschemes

Name _____ Date _____ Score / 49

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/bivariate-data-and-correlation-exam-practice

**1.** **3 marks · Paper 2 style · calculator**

The regression line of $y$ on $x$ for a bivariate dataset is given by $y = -2.5x + 10.4$. The mean of the $x$-values in the dataset is $3.2$. Find the exact mean of the $y$-values.

**2.** **3 marks · Paper 1 style · no calculator**

Four data sets have correlation coefficients $r = -0.94$, $r = 0.08$, $r = 0.62$ and $r = -0.45$.

**(a)** Write down which value shows the strongest linear correlation. **[1]**

**(b)** Write down which value shows almost no linear correlation. **[1]**

**(c)** For the data set with $r = -0.94$, describe what happens to $y$ as $x$ increases. **[1]**

**3.** **4 marks · Paper 1 style · no calculator**

For a group of cafés, the number of iced drinks sold in a day, $y$, and the maximum temperature, $x\,^\circ\text{C}$, are related by the regression line $y = 8.5x - 62$. The correlation coefficient is $r = 0.91$.

**(a)** Interpret the value $8.5$ in this context. **[1]**

**(b)** Estimate the number of iced drinks sold on a day with a maximum temperature of $24\,^\circ\text{C}$. **[2]**

**(c)** A manager says that the high temperature causes the extra sales. Comment on this claim. **[1]**

**4.** **4 marks · Paper 1 style · no calculator**

For a set of bivariate data, the regression line of $y$ on $x$ is given by $y = 3.4x - 5.2$. The mean of the $x$-values is $\bar{x} = 6.5$.

1. Find the exact mean of the $y$-values, $\bar{y}$.
2. Use the regression line to estimate the value of $y$ when $x = 8.1$.

**5.** **4 marks · Paper 2 style · calculator**

For a set of bivariate data $(x, y)$, the Pearson's product-moment correlation coefficient is $r = 0.92$. The mean of the $x$-values is $15$ and the mean of the $y$-values is $45$. The regression line of $y$ on $x$ is given by $y = 2.5x + c$.

1. Find the exact value of the constant $c$.
2. Use the regression line to predict the value of $y$ when $x = 18$.

**6.** **4 marks · Paper 2 style · calculator**

The heights, $x$ cm, and masses, $y$ kg, of seven adults are shown.

| **Height, $x$** | 150 | 155 | 160 | 165 | 170 | 175 | 180 |
|---|---|---|---|---|---|---|---|
| **Mass, $y$** | 52 | 55 | 61 | 60 | 67 | 70 | 74 |

**(a)** Find the equation of the regression line of $y$ on $x$. **[2]**

**(b)** Estimate the mass of an adult who is $172$ cm tall. **[2]**

**7.** **6 marks · Paper 2 style · calculator**

Seven pairs of measurements are recorded.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $y$ | 3.1 | 4.0 | 5.2 | 5.8 | 7.1 | 8.0 | 2.0 |

**(a)** Find the value of $r$ for all seven points. **[1]**

**(b)** The last point is found to be a recording error and is removed. Find the new value of $r$. **[2]**

**(c)** Using the six remaining points, find the regression line of $y$ on $x$ and use it to estimate $y$ when $x = 4.5$. **[3]**

**8.** **8 marks · Paper 2 style · calculator**

The table shows the number of hours, $x$, that eight students spent revising and their test score, $y$.

| **Hours, $x$** | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|
| **Score, $y$** | 45 | 50 | 58 | 61 | 70 | 72 | 80 | 86 |

**(a)** Find Pearson's product-moment correlation coefficient, $r$. **[2]**

**(b)** Describe the correlation between $x$ and $y$. **[1]**

**(c)** Find the equation of the regression line of $y$ on $x$. **[2]**

**(d)** Use your equation to estimate the score of a student who revised for $7$ hours. **[2]**

**(e)** Explain why the equation should not be used to estimate the score of a student who revised for $20$ hours. **[1]**

**9.** **6 marks · Paper 2 style · calculator**

A shop records the price, $x$ euros, of a product and the number sold, $y$, in six different weeks.

| **Price, $x$** | 10 | 14 | 18 | 22 | 26 | 30 |
|---|---|---|---|---|---|---|
| **Number sold, $y$** | 31 | 28 | 22 | 20 | 15 | 11 |

**(a)** Find the equation of the regression line of $x$ on $y$. **[2]**

**(b)** In one week the shop wants to sell $25$ items. Use the appropriate line to estimate the price it should charge. **[2]**

**(c)** Write down the value of $r$ and state what it shows. **[2]**

**10.** **7 marks · Paper 1 style · no calculator**

For a bivariate data set, the regression line of $y$ on $x$ is $y = 1.2x + 3$ and the regression line of $x$ on $y$ is $x = 0.6y - 0.5$.

a. Given that $r^2$ equals the product of the gradients of the two lines (written as $y$ on $x$ and $x$ on $y$), find the value of Pearson's correlation coefficient $r$, giving your answer exactly. **[4]**

b. Find the coordinates of the mean point. **[3]**

## Answers and worked solutions

### Question 1

3 marks

**Answer:** $2.4$

1. The regression line passes through the mean point $(\bar{x}, \bar{y})$. **M1**
2. Substitute $\bar{x} = 3.2$: $\bar{y} = -2.5(3.2) + 10.4$. **M1**
3. $\bar{y} = -8 + 10.4 = 2.4$. **A1**

### Question 2

3 marks

**Answer:** (a) $r = -0.94$; (b) $r = 0.08$

**(a)**

$-0.94$ **A1**

**(b)**

$0.08$ **A1**

**(c)**

$y$ tends to decrease **A1**

### Question 3

4 marks

**Answer:** (b) $142$

**(a)**

each extra degree Celsius goes with about $8.5$ more iced drinks sold **A1**

**(b)**

$8.5 \times 24 - 62$ **(M1)**

$142$ **A1**

**(c)**

correlation alone does not show causation **R1**

### Question 4

4 marks

**Answer:** (a) $16.9$; (b) $22.34$

1. The regression line passes through $(\bar{x}, \bar{y})$, so substitute $x = 6.5$. **M1**
   $\bar{y} = 3.4(6.5) - 5.2 = 22.1 - 5.2 = 16.9$. **A1**
2. Substitute $x = 8.1$ into the regression line:
   $y = 3.4(8.1) - 5.2$. **M1**
   $y = 27.54 - 5.2 = 22.34$ (accept $22.3$). **A1**

### Question 5

4 marks

**Answer:** (a) $c = 7.5$; (b) $52.5$

1. The regression line passes through $(\bar{x}, \bar{y}) = (15, 45)$; substitute:
   $45 = 2.5(15) + c$. **M1**
   $45 = 37.5 + c \implies c = 7.5$. **A1**
2. Substitute $x = 18$ into $y = 2.5x + 7.5$:
   $y = 2.5(18) + 7.5$. **M1**
   $y = 45 + 7.5 = 52.5$. **A1**

### Question 6

4 marks

**Answer:** (a) $y = 0.729x - 57.5$; (b) $67.8$ kg

**(a)**

$y = 0.729x - 57.5$ **A1A1**

**(b)**

$0.7285\ldots \times 172 - 57.5$ **(M1)**

$67.8$ kg **A1**

### Question 7

6 marks

**Answer:** (a) $r = 0.237$; (b) $r = 0.997$; (c) $6.52$, using $y = 0.983x + 2.09$

**(a)**

$r = 0.237$ **A1**

**(b)**

six points **(M1)**

$r = 0.997$ **A1**

**(c)**

$y = 0.983x + 2.09$ **A1A1**

$6.52$ **A1**

### Question 8

8 marks

**Answer:** (a) $r = 0.998$; (c) $y = 3.93x + 37.7$; (d) $65.2$

**(a)**

$r = 0.998$ **A2**

**(b)**

strong positive **A1**

**(c)**

$y = 3.93x + 37.7$ **A1A1**

**(d)**

$3.934\ldots \times 7 + 37.70\ldots$ **(M1)**

$65.2$ **A1**

**(e)**

$20$ is outside the range of the data (extrapolation)
**R1**

### Question 9

**6 marks**

**Answer:** (a) $x = -0.983y + 40.8$; (b) €$16.2$; (c) $r = -0.995$

**(a)**

$x = -0.983y + 40.8$ **A1A1**

**(b)**

use $x$ on $y$ with $y = 25$ **(M1)**

€$16.2$ **A1**

**(c)**

$r = -0.995$ **A1**

very strong negative linear correlation **A1**

### Question 10

**7 marks**

**Answer:** (a) $r = 3\sqrt{2}/5$; (b) $(65/14, 60/7)$

a. $r^2 = 1.2 \times 0.6 = 0.72$ $M$**1A1**

$r = \sqrt{0.72} = \dfrac{3\sqrt{2}}{5}$ (positive since both gradients are positive) **A1R1**

b. $x = 0.6(1.2x + 3) - 0.5$ **M1**

$0.28x = 1.3 \implies \bar{x} = \dfrac{65}{14}$ **A1**

$\bar{y} = 1.2 \times \dfrac{65}{14} + 3 = \dfrac{60}{7}$ **A1**

Every solution is also on ibmathrevision.com/slaa/bivariate-data-and-correlation-exam-practice
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