Binomial Theorem: exam practice

10 questions · 50 marks · worked solutions and markschemes

Name _____ Date _____ Score / 50

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/slaa/binomial-theorem-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Find the first three terms, in ascending powers of x, in the expansion of $(3+x)^4$.

2. **4 marks · Paper 1 style · no calculator**

The coefficient of x^2 in the binomial expansion of $(1+kx)^5$ is 40. Given that $k>0$, find the exact value of k.

3. **4 marks · Paper 1 style · no calculator**

In the binomial expansion of $\left(\frac{x}{2}+\frac{2}{x}\right)^6$, find the exact value of the constant term (the term independent of x).

4. **5 marks · Paper 1 style · no calculator**

In the expansion of $(3+px)^6$, the coefficient of the x^4 term is four times the coefficient of the x^2 term. Find the possible values of p.

5. **5 marks · Paper 1 style · no calculator**

In the expansion of $(x+3)^n$, where $n \in \mathbb{Z}^+$, the coefficient of x^{n-2} is 324.

Find the value of n. **[5]**

6. **6 marks · Paper 2 style · calculator**

Consider the expansion of $\left(2x+\frac{k}{x}\right)^9$, where $k>0$. The coefficient of the term in x^3 is equal to the coefficient of the term in x^5. Find the exact value of k.

7. **4 marks · Paper 1 style · no calculator**

Consider the expansion of $(1+x)^6$.

a. Write down the first three terms, in ascending powers of x. **[2]**

b. Hence find an estimate for 1.02^6. **[2]**

8. **5 marks · Paper 1 style · no calculator**

Consider the expansion of $\left(2x^2 - \frac{1}{x}\right)^6$.

(a) Write down the number of terms in this expansion. **[1]**

(b) Find the term independent of x. **[4]**

9. **6 marks · Paper 1 style · no calculator**

(a) Find the first three terms, in ascending powers of x, in the expansion of $(2-x)^5$. **[3]**

(b) Hence find the coefficient of x^2 in the expansion of $(1+3x)(2-x)^5$. **[3]**

10. **8 marks · Paper 2 style · calculator**

In the expansion of $(1+ax)^n$, where $n \in \mathbb{Z}^+$, the coefficient of x is -20 and the coefficient of x^2 is 180.

(a) Show that $a(n-1) = -18$. **[3]**

(b) Find the value of n and the value of a. **[3]**

(c) Find the coefficient of x^3. **[2]**

Answers and worked solutions

Question 1

3 marks

Answer: $81 + 108x + 54x^2$

$(3+x)^4 = \binom{4}{0}3^4 + \binom{4}{1}3^3(x) + \binom{4}{2}3^2(x)^2 + \ldots$ (M1)

$= 1(81) + 4(27)(x) + 6(9)(x^2)$ (A1)

$= 81 + 108x + 54x^2$ A1

Question 2

4 marks

Answer: $k = 2$

Write the x^2 term: $\binom{5}{2}(1)^3(kx)^2 = 10k^2x^2$ M1

Set the coefficient equal to 40: $10k^2 = 40$ M1

Solve: $k^2 = 4$ (A1)

$k = 2$ (since $k > 0$) A1

Question 3

4 marks

Answer: 20

Write the general term: $\binom{6}{r}\left(\frac{x}{2}\right)^{6-r}\left(\frac{2}{x}\right)^r$ M1

Set the power of x to zero:

$(6-r) - r = 0 \implies r = 3$ A1

Substitute $r = 3$: $\binom{6}{3}\left(\frac{1}{2}\right)^3(2)^3$ (M1)

$= 20 \times \frac{1}{8} \times 8 = 20$ A1

Question 4

5 marks

Answer: $p = 6, p = -6$

Write down the x^2 term:

$\binom{6}{2}3^4(px)^2 = 15(81)p^2x^2 = 1215p^2x^2$. (A1)

Write down the x^4 term:

$\binom{6}{4}3^2(px)^4 = 15(9)p^4x^4 = 135p^4x^4$. (A1)

Set the x^4 coefficient equal to four times the x^2 coefficient: $135p^4 = 4(1215p^2)$. (M1)

Rearrange and factorise:

$135p^4 = 4860p^2 \implies p^4 - 36p^2 = 0 \implies p^2(p^2-36) = 0$. (M1)

$p = 0$ makes both coefficients zero (trivial case), so $p = \pm 6$. A1

Question 5

5 marks

Answer: 9

term in x^{n-2} is $\binom{n}{2}x^{n-2}3^2$ (M1)

$9\binom{n}{2} = 324$, so $\binom{n}{2} = 36$ A1

$\frac{n(n-1)}{2} = 36$, so $n^2 - n - 72 = 0$ M1

$(n-9)(n+8) = 0$ (A1)

$n = 9$ (as $n > 0$) A1

Question 6

6 marks

Answer: $k = 6/7$

General term:

$\binom{9}{r}(2x)^{9-r}(kx^{-1})^r = \binom{9}{r}2^{9-r}k^rx^{9-2r}$. (M1)

For x^3: $9 - 2r = 3 \implies 2r = 6 \implies r = 3$.

Coefficient $=$

$\binom{9}{3}2^6k^3 = 84 \times 64 \times k^3 = 5376k^3$. (A1)

For x^5: $9 - 2r = 5 \implies 2r = 4 \implies r = 2$.

Coefficient $=$

$\binom{9}{2}2^7k^2 = 36 \times 128 \times k^2 = 4608k^2$. (A1)

Equating coefficients: $5376k^3 = 4608k^2$. (M1)

Since $k > 0$, divide by k^2:

$5376k = 4608 \implies k = \frac{4608}{5376}$. (A1)

$k = \frac{6}{7}$. A1

Question 7

4 marks

Answer: (a) $1 + 6x + 15x^2$; (b) 1.126

a. $1 + 6x + 15x^2$ A2

b. $x = 0.02$ (M1)

$1 + 0.12 + 0.006 = 1.126$ A1

Question 8

5 marks

Answer: (a) 7; (b) 60

(a)

7 A1

(b)

General term $\binom{6}{r}(2x^2)^{6-r}\left(-\frac{1}{x}\right)^r$ (M1)

Power of x: $2(6-r) - r = 0$, so $r = 4$ (A1)

$\binom{6}{4}(2x^2)^2\left(-\frac{1}{x}\right)^4 = 15 \times 4x^4 \times \frac{1}{x^4}$ (M1)

$= 60$ A1

Question 9

6 marks

Answer: (a) $32 - 80x + 80x^2$; (b) -160

(a)

$2^5 + \binom{5}{1}2^4(-x) + \binom{5}{2}2^3(-x)^2$ **(M1)**

$32 - 80x + 80x^2$ **A2**

(b)

$1 \times 80x^2 + 3x \times (-80x)$ **(M1)(A1)**

$80 - 240 = -160$ **A1**

Question 10

8 marks

Answer: (b) $n = 10,\ a = -2$; (c) -960

(a)

$na = -20$ **A1**

$\binom{n}{2}a^2 = \frac{n(n-1)}{2}a^2 = 180$ **A1**

Divide: $\frac{n(n-1)a^2}{2na} = \frac{180}{-20}$, so

$\frac{(n-1)a}{2} = -9$ **M1**

$a(n-1) = -18$ **AG**

(b)

$na - a = -18$ with $na = -20$ gives

$-20 - a = -18$ **(M1)**

$a = -2$ **A1**

$n = 10$ **A1**

(c)

$\binom{10}{3}(-2)^3$ **(M1)**

$= -960$ **A1**

Every solution is also on ibmathrevision.com/slaa/binomial-theorem-exam-practice

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